

A UNIVARIATE NONLINEAR MODEL OF THE RETURNS ON ISTANBUL STOCK EXCHANGE 100 INDEX

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Abstract: Asymmetric behaviors are common in economics and finance. Since it is not possible to capture asymmetric behaviors by linear models, nonlinear models are developed in order to explain asymmetric behaviors exhibited by such time series. Findings in this study show that ISE 100 index's behavior cannot be estimated by linear univariate models for the period after 2000. Therefore, it is our aim to construct and estimate nonlinear time series models of ISE 100 index. The results obtained also confirm that ISE 100 index exhibits nonlinear behavior.

Key words: ISE 100, nonlinear time series, univariate time series.

JEL Codes: G15, G11, C22.

İSTANBUL MENKUL KIYMETLER BORSASI 100 ENDEKSİNİN DEK DEĞİŞKENLİ DOĞRUSAL OLMAYAN BİR MODELİ

Özet: Ekonomi ve finans alanında asimetrik davranış yaygın olarak gözlemlenir. Asimetrik davranışı doğrusal modellerle modellemek olanaklı olmadığından, bu tür zaman serilerinin sergilediği asimetrik davranışı açıklamak için uygun doğrusal olmayan modeller geliştirilir. Bu çalışmanın bulguları İMKB 100 endeksinin 2000 yılı sonrasında sergilediği davranışının doğrusal modeller ile tahmin edilemeyeceğini göstermektedir. Bu nedenle bu çalışmanın amacı İMKB 100 endeksinin zaman serisinin doğrusal olmayan bir modelini kurgulamak ve tahmin etmektir. Kurgulanan ve hesaplanan modelin sonuçları İMKB 100 endeksinin doğrusal olmayan bir davranış sergilediğini doğrulamaktadır.

Anahtar kelimeler: İMKB 100, doğrusal olmayan zaman serileri, dek değişkenli zaman serileri

I. INTRODUCTION

Stock exchange markets provide an important channel for transferring savings into real economy. However, there are other alternative and competing financial markets to stock exchange markets. To attract more savings, each of these markets must provide higher returns than their competing ones. In addition, stability and predictability of returns are important factors for savers in their decision on channeling their savings. Stability and predictability of asset prices together determine riskiness of portfolios. As pointed out by Schwert (2011), it is widely agreed that volatility should be measured in percentage

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change in prices, or rates of returns. Furthermore, large swings in returns in financial markets are among the leading indicators of the condition of the whole economy. Therefore, empirical analysis of how returns behave will provide important information for savers, investors, and policy makers. On the other hand, as mentioned by Ince (2005), financial markets are complex, nonstationary and deterministically chaotic systems. Thus, a linear behavior assumption may lead to choosing inappropriate techniques in modeling financial markets. That is specifically correct for financial markets in emerging countries such as Turkey because financial markets in those countries are more vulnerable to domestic as well as international economic developments. In emerging economies, in addition to macroeconomic variables such as real economic activity, money supply, inflation, exchange rates, oil prices, and current account deficits, the political risks also affect return on stock markets. Hence, in those countries a true prediction of the behavior of stock return is necessary.

Estimating the behavior of stock returns from time series with high frequencies has received considerable attention. However, time series with high frequencies, including financial series, may display asymmetric behavior. Most of the time, this asymmetry is surmised as random behavior. Kocenda and Cerny (2007) explain this case as follows:

Chaotic systems of dimensionality can generate seemingly random numbers that may give an impression of white noise thereby hiding their true nature. Under presumed randomness, a nonlinear pattern can hide without being detected. Exchange rates, stock market returns, and other macroeconomic variables of generally high frequency, for example, may originate from low-dimensional chaos. (p.118)

Hence, the purpose of this study is to establish whether the return on ISE 100 is generated through a nonlinear process, and to model the series with appropriate techniques if the data generating process is not linear. We utilize BDS (Brock, Dechert, and Scheinkman), Keenan, and Tsay tests to determine whether the return is linear. After testing the series, we use DTARCH (Double Threshold Autoregressive Conditional Heteroscedasticity) in modeling the series.

II. ECONOMETRIC METHODOLOGY

In this subsection we briefly explain the tests utilized in determining whether the return on ISE 100 is linear. Both nonparametric and parametric tests are used for this purpose. The BDS test is a nonparametric test, while Keenan and Tsay tests are parametric tests. Finally in this section we elaborate on DTARCH process used in modeling the series.

BDS Test: Brock, Dechert, and Scheinkman (1987) developed the so-called BDS test on the basis of the assumption that there cannot be hidden (nonlinear) patterns in a purely random independent and identically distributed

(iid) process. BDS test is a nonparametric test with null hypothesis that the series is iid. Since this procedure is based on whether the residuals are iid, this test can be used for model specification (Kocenda and Cerny, 2007).

According to Tsay (2005, 185), the basic idea of BDS test is to make use of ‘correlation integral’ which is a popular concept of chaotic time series analysis. This integral was defined by Grassberger and Procaccio (1983), and as pointed out by Kocenda and Cerny (2007, p.120), it is unique in revealing nonlinearity without being affected by the linear dependency in the series.

Given k-dimensional time series X_t and observations $\{X_t\}_{t=1}^{T_k}$, Tsay (2005, p.185) defines this integral as follows:

$$C_k(\delta) = \lim_{T_k \rightarrow \infty} \frac{2}{T_k(T_k - 1)} \sum_{i < j} I_\delta(X_i, X_j) \tag{1}$$

where $I_\delta(u, v)$ is an indicator variable that equals one if $\|u - v\| < \delta$ and zero otherwise, and $\|\cdot\|$ is sup-norm. Correlation integral measures the fraction of pairs of observations within δ tolerance distance. In other words, if the distance between two observations in the k-dimensional series is less than δ , indicator becomes 1, and correlation integral sums this value over all pairs in order to measure its ratio in X_t observations.

k-dimensional series X_t and observations $\{X_t\}_{t=1}^{T_k}$ in Tsay’s (2005) correlation integral definition indicate that the original time series is embedded in k dimension. In other words, the observations $\{X_t\}_{t=1}^{T_k}$ are no longer scalar but vectors of k dimension. In order to check whether $X = (x_1, x_2, x_3, \dots, x_N)$ series is linear, we transform each consecutive k observations into k dimensional vectors and obtain X_t^k series in which each observation is a vector itself. Observations of this new series, which are referred to in $\{X_t^k\}_{t=1}^{T_k}$ form, can be explicitly expressed as follows:

$$\begin{aligned} x_1^k &= (x_1, x_2, \dots, x_k) \\ x_2^k &= (x_2, x_3, \dots, x_{k+1}) \\ &\vdots \\ x_{N-k}^k &= (x_{N-k}, x_{N-k+1}, \dots, x_N) \end{aligned}$$

where T_k shows the number of observations in X_t series. The number of data points of the newly constructed k-dimensional series is $T_k = N - k$, N is the number of observations in the original series, and K is the dimension of each data point in the new series.

After the new series embedded in k - dimension is constructed, observations are paired with each other such that each observation is paired with one of the remaining observations once. Then whether a pair is in δ tolerance distance is established. In this formulation, the correlation integral measures the fraction of pairs within δ tolerance distance.

If the series consists of observations that are iid random variables, then correlation integral should satisfy the relation $C_k(\delta) = C_1(\delta)^k$. A deviation from this relationship means that the observations in the series are not iid. Utilizing this feature, the test statistic of the BDS test is defined as follows:

$$BDS_I(\delta, T) = \frac{T^{1/2} [C_k(\delta, T) - C_1(\delta, T)^k]}{\sigma_k(\delta, T)} \quad (2)$$

$\sigma_k(\delta, T)$ term in equation (2) is the standard deviation of the series generated by the $C_k(\delta, T) - C_1(\delta, T)^k$ operation in the numerator. The test statistic has a standard normal limiting distribution.

Keenan Test: Keenan (1985) adapted Tukey's one degree of freedom for nonadditivity test in order to test for linearity versus second-order Volterra expansion. "The test is designed to have optimal local power against departure from the linear autoregressive function in the direction of the square of the linear autoregressive function" (Chan, 2012, p.36). According to Tsay (2005), this method is similar to RESET, but it is modified by including squares of independent variables as dependent variable in auxiliary regression and eliminating linear dependency between estimated squared term and lagged terms.

Keenan (1985, p.39) states Volterra expansion, which is a common form in nonlinear stationary time series, as follows:

$$Y_t = \mu + \sum_{u=-\infty}^{\infty} a_u \varepsilon_{t-u} + \sum_{u,v=-\infty}^{\infty} a_{uv} \varepsilon_{t-u} \varepsilon_{t-v} + \sum_{u,v,w=-\infty}^{\infty} a_{uvw} \varepsilon_{t-u} \varepsilon_{t-v} \varepsilon_{t-w} + \dots \quad (3)$$

where $\{\varepsilon_t, -\infty < t < \infty\}$ is strictly stationary and an iid process with zero mean. Keenan (1985, 40) takes linearity as the absence of multiplicative terms in equation 3. Therefore the test of linearity is to test whether the multiplicative terms in equation 3 is statistically significant. Accordingly the null hypothesis of the test is $\hat{\eta}_0 = 0$, where $\hat{\eta}_0$ is the regression coefficient of the auxiliary regression of two residuals (Tsay, 2005, p.187).

For a fixed M , Keenan (1985, p.41) recounts the steps of the procedure as follows:

Regress Y_s on $\{1, Y_{s-1}, \dots, Y_{s-M}\}$ and calculate fitted values $\{\hat{Y}_s\}$ and residuals $\{\hat{e}_s\}$ for $s = M+1, \dots, n$, and the residual sum of squares $\langle \hat{e}, \hat{e} \rangle = \sum \hat{e}_s^2$.

Regress \hat{Y}_s on $\{1, Y_{s-1}, \dots, Y_{s-M}\}$ and calculate residuals $\{\hat{\xi}_s\}$ for $s = M+1, \dots, n$.

Regress $\hat{e} = (\hat{e}_{M+1}, \dots, \hat{e}_n)$ on $\hat{\xi} = (\hat{\xi}_{M+1}, \dots, \hat{\xi}_n)$ and obtain $\hat{\eta}$ and \hat{F} via

$$\hat{\eta} = \hat{\eta}_0 \left(\sum_{t=M+1}^n \hat{\xi}_t^2 \right)^{1/2}$$

where $\hat{\eta}_0$ is regression coefficient, and

$$\hat{F}_{1, n-2M-2} = \frac{\hat{\eta}^2 (n-2M-2)}{\langle \hat{e}, \hat{e} \rangle - \hat{\eta}^2}$$

where the degrees of freedom associated with $\langle \hat{e}, \hat{e} \rangle$ are $(n-M)-M-1$.

Tsay Test: Tsay (1986) developed another test which "...retains the simplicity Keenan test has, yet it is considerably more powerful" (Tsay, 1986, p.462). This new test dubbed Tsay test is also base on Volterra expansion. In the Volterra expansion expressed in (3), if we take μ as mean, and the terms $\{a_{uv}\}, \{a_{uvw}\}, \dots$ different from zero, then Y_t is not linear.

Using \hat{Y}_t^2 in the second step in Keenan test procedure only requires 1 degree of freedom and it is useful in small samples (Tsay, 1986, p.462). On the other hand, in medium and large samples, if we use the modified second step instead of summed \hat{Y}_t^2 , the power of the test should increase.

The steps of the procedure in Tsay test is as follows (1986, p.462):

Regress Y_t on $\{1, Y_{t-1}, \dots, Y_{t-M}\}$ by least square method and obtain the residuals $\{\hat{e}_s\}$ for $t = M+1, \dots, n$. The regression model will be denoted by

$$Y_t = W_t \Phi + e_t \tag{4}$$

where $W_t = (1, Y_{t-1}, \dots, Y_{t-M})$ and $\Phi_t = (\Phi_0, \Phi_{t-1}, \dots, \Phi_{t-M})^T$ with M being a prespecified positive integer, n sample size, and the superscript T denoting the matrix transpose.

Regress Z_t vector on $\{1, Y_{t-1}, \dots, Y_{t-M}\}$ and obtain the residual vector $\{\hat{X}\}$ for $t = M+1, \dots, n$. Here the multivariate regression model is

$$Z_t = W_t H + X_t$$

where Z_t is an $m = \frac{1}{2}M(M+1)$ dimensional vector defined as

$Z_t^T = \text{vech}(U_t^T U_t)$ where $U_t = (Y_{t-1}, \dots, Y_{t-M})$ and vech denotes the half stacking vector. In other words Z_t^T is obtained from the symmetric matrix $U_t^T U_t$ by the usual column stacking operator but using only the elements on or below the main diagonal of each column.

Regress \hat{e}_t on \hat{X}_t and let \hat{F} be the ratio of the mean square of regression to the mean error. That is, fit

$$\hat{e}_t = \hat{X}_t \beta + \varepsilon_t \quad (t = M+1, \dots, n) \quad (5)$$

and define

$$\hat{F} = \frac{\left\{ \left(\sum \hat{X}_t \hat{e}_t \right) \left(\sum \hat{X}_t^T \hat{X}_t \right)^{-1} \left(\sum \hat{X}_t^T \hat{e}_t \right) / m \right\}}{\left\{ \sum \hat{\varepsilon}_t^2 / (n - M - m - 1) \right\}}$$

where the summations over t from $M+1$ to n and $\hat{\varepsilon}_t$ is the least squares residual for (5).

The null hypothesis of the Tsay test is that the series are generated by an AR process. “The AR order, if missing, is estimated by minimizing AIC via ar function, i.e. fitting autoregressive model to the data. The default fitting method of the ar function is ‘Yule-Walker’ ” (Chan, 2012, p.69).

The DTARCH Model: As the name suggests DTARCH (Double Threshold ARCH) has threshold in both mean and variance. Therefore DTARCH process captures the asymmetric behaviors in conditional mean and conditional variance together. In a way DTARCH process is equivalent to modeling the conditional variance with TARCH (Threshold ARCH) and conditional mean with SETAR (Self-Exciting Threshold AR) processes simultaneously.

SETAR process is fundamentally an augmentation of AR model. If a series follows a process where it cannot be modeled with a single ar function throughout the series, such that above or below certain values it is modeled with a different AR processes this indicates that the series is generated by a TAR (Threshold Autoregressive) process. The values that assort the observations according to the AR process they follow are called thresholds. Thresholds separate the series into regimes. Each AR process that forms the TAR model is called a regime. A TAR model has at least two regimes. This means it is possible for a series to have one threshold as well as many. Furthermore, the

thresholds can be established using another series or the original series itself. If a different series is used to determine the thresholds, the process is named TAR. When the thresholds are established by using the series itself, the process is called SETAR. In other words Self-Exciting term in a TAR process indicates whether the observations of the series itself are employed while the values for thresholds are searched. Stigler (2012, 8) depicts a general TAR process as follows

$$Y_t = \begin{cases} \mu_1 + \rho_{1,1}Y_{t-1} + \dots + \rho_{1,p1}Y_{t-p1} + \varepsilon_t & X_{t-d} \geq \theta_{m-1} \\ \mu_2 + \rho_{2,1}Y_{t-1} + \dots + \rho_{2,p2}Y_{t-p2} + \varepsilon_t & \theta_{m-1} \geq X_{t-d} \geq \theta_{m-2} \\ \dots & \theta_{..} \geq X_{t-d} \geq \theta_{..} \\ \mu_m + \rho_{m,1}Y_{t-1} + \dots + \rho_{m,pm}Y_{t-pm} + \varepsilon_t & \theta_1 \geq X_{t-d} \end{cases}$$

where the parameters of the process is listed as below

m : the number of regimes

$\mu_1 \dots \mu_m$: the intercepts in each regime

$p_{j,1} \dots p_{j,m-1}$: the number of lags in regime j

$\theta_1 \dots \theta_{m-1}$: the thresholds

d : the delay of the transition variable

X_{t-d} : the transition variable

The transition variable indicates the series employed for establishing the thresholds. If the transition variable for the specified d the delay value is chosen as the original series $X_{t-d} = Y_{t-d}$, then the model is called SETAR.

ARCH process is developed by Engel (1982). Let X_t and Y_t be two time series and Ω_t be the information set, first order ARCH process is depicted as below:

$$Y_t = a + \beta X_t + \varepsilon_t$$

$$\varepsilon_t | \Omega \square iid \quad N(0, h_t)$$

$$h_t = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2$$

Furthermore, for a q -order ARCH process only the last equation changes into:

$$h_t = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 + \gamma_2 \varepsilon_{t-2}^2 + \dots + \gamma_q \varepsilon_{t-q}^2 = \gamma_0 + \sum_{j=1}^q \gamma_j \varepsilon_{t-j}^2$$

The idea that both SETAR and ARCH processes may be observed in the same series originates from Tong (1993). This process is named SETAR-

ARCH, since only the conditional mean has a threshold. Let $\{\varepsilon_t\}$ has an iid standard normal, $\gamma > 0$, $\phi_i \geq 0$ and V_t have the form

$$V_t = \phi_0 + \sum_{i=1}^q \phi_i X_{t-i}^2,$$

and finally let the terms of f be piecewise linear; Tong (1993, p.116) defines the SETAR-ARCH model as follows:

$$X_t = f(X_{t-1} + \dots + X_{t-k}) + \varepsilon_t \sqrt{V_t}.$$

Although SETAR-ARCH model combines the advantages of the SETAR model and the ARCH model, it assumes a fixed description of the conditional variance (Li and Li, 1996). Conditional variance may display behaviors that are asymmetric as well, and therefore cannot be modeled by SETAR-ARCH (Baragona and Battaglia, 2006, p.443). In other words SETAR-ARCH model cannot model asymmetric behavior of the conditional variance successfully. SETAR-ARCH process models the conditional variance so that it consists of a single regime when in fact the series may be generated by a process where the conditional variance is separated into several regimes. Such a model where both conditional mean and conditional variance consist of several regimes separated by thresholds is developed by Li and Li (1996) and Liu, Li, and Li (1997). These are the DTARCH models mentioned at the beginning of this section. The time series $\{X_t\}$ generated with a DTARCH process may be modeled as follows:

$$X_t = \phi_0^{(u)} + \sum_{i=1}^{p_u} \phi_i^{(u)} X_{t-i} + \varepsilon_t \quad X_{t-d} \in R_u, \quad u = 1, \dots, k$$

where ε_t term is the white noise process with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma_\varepsilon^2$ and $h_t = \text{Var}(X_t | X_{t-1}, X_{t-2}, \dots)$ is the conditional variance which is written as

$$h_t = \gamma_0^{(v)} + \sum_{i=1}^{q_v} \gamma_i^{(v)} \varepsilon_{t-i}^2 \quad \varepsilon_{t-c} \in R_v^*, \quad v = 1, \dots, h$$

in an explicit form. The parameters in this process are as follows:

u: the number of regimes in the SETAR part of the model

v: the number of regimes in the TARCH part of the model

d: the delay of the transition variable in the SETAR part of the model

c: the delay of the transition variable in the TARCH part of the model

p_1, \dots, p_k : The AR order in the SETAR part of the model

q_1, \dots, q_h : The ARCH order in the TARCH part of the model

R_u, R_v^* : transition variable for the SETAR and TARCH parts of the model respectively.

A DTARCH model with two regimes in both mean and variance or a DTARCH $(p_1, p_2; q_1, q_2)$ model may be displayed as follows:

$$X_t = \begin{cases} \Phi_0^{(1)} + \sum_{i=1}^{p_1} \Phi_i^{(1)} X_{t-i} + \varepsilon_t, & X_{t-d} \leq r \\ \Phi_0^{(2)} + \sum_{i=1}^{p_2} \Phi_i^{(2)} X_{t-i} + \varepsilon_t, & X_{t-d} > r \end{cases}$$

$$h_t = \begin{cases} \gamma_0^{(1)} + \sum_{i=1}^{q_1} \gamma_i^{(1)} \varepsilon_{t-i}^2, & X_{t-c} \leq r \\ \gamma_0^{(2)} + \sum_{i=1}^{q_2} \gamma_i^{(2)} \varepsilon_{t-i}^2, & X_{t-c} > r \end{cases}$$

In order to display this model; the same parameters mentioned previously for the compact form of DTARCH are used. The simple explicit display of DTARCH $(p_1, p_2; q_1, q_2)$ above can be generalized for a DTARCH $(p_1, p_2, \dots, p_m; q_1, q_2, \dots, q_m)$ model. Additionally the model may also be reduced so that it may explain simpler processes. For example, the model turns into a SETAR-ARCH model when the conditional variance is modeled with a single regime. In other words

$$DTARCH(p_1, p_2, \dots, p_m; q) = SETAR-ARCH(p_1, p_2, \dots, p_m).$$

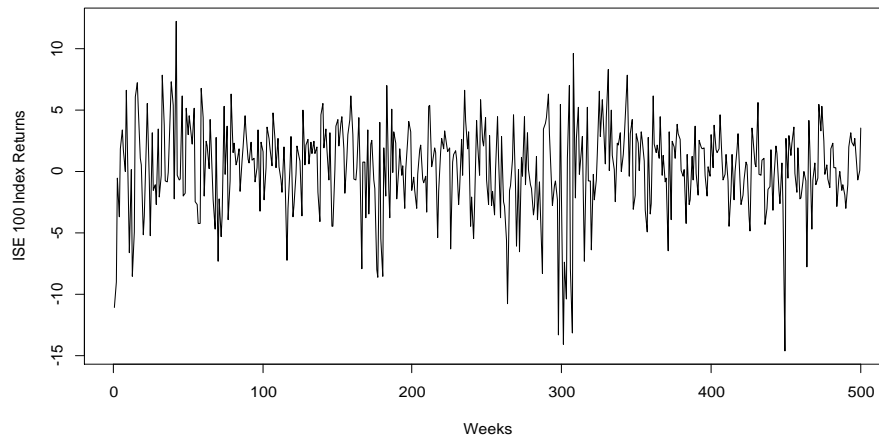
DTARCH model requires two sets of thresholds to be discovered. This raises cost of the computational resources and complicates the estimation procedure. Baragona and Cucina (2008) propose a method based on genetic algorithms. This method utilizes genetic algorithms in order to discover structural parameters such as the threshold and the order of ARCH and AR processes in each regime.

III. DATA AND EMPIRICAL RESULTS

This study aims to model the returns on ISE 100 index after 2002. The daily closing values of the index are obtained from the online database of Central Bank of the Republic of Turkey (TCMB) for the period between December 20, 2002, and August 3, 2012. Rather than investigating a high frequency series in a short period of time, the study aims to lower the frequency and scrutinize a longer period thus the general properties of the ISE 100 index may be established. Therefore, the average of the daily closing prices is calculated in order to obtain weekly return series.

Let R_t be the returns and P_t be the weekly average of closing prices of ISE 100 index, then the returns is calculated by $R_t = 100 * (\ln P_t - \ln P_{t-1})$ equation. This new series is consists of 450 observations and is depicted graphically in Figure 1.

Figure 1: Returns on ISE 100 Index



Stationarity of the returns series is tested with the Philips-Perron (PP) unit root test. The reason PP test is employed is that although it is based on Augmented Dickey-Fuller (ADF) test; it is also autocorrelation and heteroscedasticity robust. The result of the test (test statistic = -18.8547 and p-value: 0.01) shows that the returns on ISE 100 is stationary.

The linearity of the series is tested using BDS, Keenan and Tsay tests. BDS test is a nonparametric test and based on whether two pairs in a series embedded in k-dimension is within a specified distance. The BDS test is conducted for the embedding dimension two through four. The neighborhood values around the closed points are 1.7247, 3.4493, 5.1740 and 6.8987. The neighborhood values are determined by multiplying the standard deviation of the weekly average of returns on the ISE 100 index with 0.5, 1, 1.5 and 2 respectively.

The test statistics of the BDS test is depicted in Table 1. The first row of Table 1 contains neighborhood values for the closed points in bracket. The first column consists of embedding dimension. The test statistics regarding the neighborhood values for the closed points is computed for each embedding dimension. Since test statistic has standard normal distribution the computed test values is compared with this distribution and the pertinent p-value is written besides the test value.

Table 1: *BDS test statistics*

	[1.7247]	[3.4493]	[5.1740]	[6.8987]
[2]	3.3896 (0.0007)	3.7275 (0.0002)	3.9715 (0.0001)	4.7517 (0.0000)
[3]	4.4188 (0.0000)	4.8960 (0.0000)	5.0201 (0.0000)	5.5920 (0.0000)
[4]	4.5320 (0.0000)	5.1088 (0.0000)	5.2329 (0.0000)	6.0085 (0.0000)
[5]	3.5386 (0.0004)	5.9942 (0.0000)	5.2336 (0.0000)	6.1297 (0.0000)
[6]	3.5971 (0.0040)	5.4386 (0.0000)	5.5640 (0.0000)	6.3298 (0.0000)

The results of BDS test indicates that the series is not linear. The result of Keenan test (test statistic: 1. 970989 and p-value: 0.161) denotes that the null hypothesis of linearity cannot be rejected. However Tsay test, which is based on Keenan test but more powerful, has the test statistic value of 3.136 (p-value: 0.0007) and thus rejects linearity hypothesis. Among BDS, Keenan and Tsay tests Both BDS and Tsay tests acknowledges that the series is nonlinear. Furthermore Keenan test the only test that cannot reject linearty is considerably less powerful than Tsay test. Therefore it is concluded that the series cannot be modelled with a linear process.

Table 2: *DTARCH (3,2;2,3) model on returns on ISE 100 Index*

Parameters	Regime 1	Regime 2
Phi(0)	-0.740988	0.283235
Phi(1)	-0.014504	0.193909
Phi(2)	-0.194608	0.076709
Phi(3)	0.223210	-
Sigma(0)	8.801216	6.481065
Sigma(1)	1.323428	0.135555
Sigma(2)	0.154899	0.066662
Sigma(3)	-	0.190085

Taking the asimetric behavior of the conditional variances of the series in financial markets into consideration the returns series is modelled as depicted in Table 2. The series is modelled with as a DTARCH (3,2;2,3) process where different threshold values are chosen for conditional mean and conditional part of the models. Conditional mean and variance both exhibit two regimes. The threshold value of the conditional mean of the model is -1.901567 and the

threshold value of the conditional variances is -1.352475 . In the SETAR part of the model first regime consists of 151 observations, the second regime consists of 346 observations. In the TARCH part of the model first regime consists of 135 observations, the second regime consists of 362 observations.

The Portmanteau test, which checks whether autocorrelation exists between the residual terms of the model, has the following test statistics: (for conditional mean) $Q_m = 11.09040$ and (for conditional variance) $Q_{mm} = 8.16314$ for up to ten lags. These results indicate that up to ten lags there is no autocorrelation in the residuals.

IV. CONCLUSION

This study concludes that returns on ISE 100 index is stationary and can be modeled by nonlinear processes. The nonlinearity of the series is tested with three tests where two test indicated nonlinearity. Though Keenan test could not reject linearity, the other two rejected linearity and one of them is a more powerful version of the Keenan test. Consequently the returns on ISE 100 index is modeled with DTARCH (3,2;2,3) process.

This information is very important in both forecasting behavior of stock exchange markets and policy formulation on the basis of that forecasting. A linear behavior assumption may lead to inappropriate policy design in channeling savings into real economy, and therefore misallocation of limited resources. A nonlinear behavior naturally leads to further risks, which must be taken into account in decision making by both market participants and policy makers.

The nonlinear behavior is captured through thresholds in both the conditional mean and variance. Both parts of the model exhibit two regimes. SETAR part of the model presents two very different regimes. While economy is in the first regime (below the threshold) return values has a tendency to switch to the second regime. The second regime is more stable, parameters in are all positive, indicating that former return values have positive effect on the current return. The TARCH part of the model indicates that first regime depicts more volatility relative to the second regime. This leads to the conclusion that although return values has a tendency to be above -1.901567 , the return values below -1.352475 indicate high volatility which may prolong the switch to the second regime. On the other hand when the return prices are at second regime for conditional variance or above -1.352475 then, although returns may not be positive, return values tend to be less volatile and remain in this regime.

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