Some Examples of Formation of Shells and Their Role in Establishment of Equilibrium

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Abstract
In this article we discuss the concept of equilibrium establishment in four most usual instances as is electrostriction and vaporization as related to the concept of equilibrium shell formation. Two more cases are then studied which are of relevance. One is the Brownian movement, the study of which is essential for pedagogical reasons as to exclusion of basic facts from statistics in a simple manner. The last one case studied is the very passage of current in a conductor. What is new in all these discussions is a simple but more detailed approach connecting the micro world to simple models of the macro world so that new comprehension is achieved. We will discuss matters in terms of order of magnitude of the quantities on what is to follow.

Keywords: Polarization, surface, Bernoulli principle, electricity.

First Case: Electrostriction

The average charge due to friction is 10^{-9}Cb=1nCb. The charge of the electron is 1e=10^{-19} Cb. Therefore totally 10^{10} charged particles are being produced. If they are thought of as creating a monoatomic layer – which is not exactly true for a glass rod or any insulating material for which polarization comes into play but the approximation may be seen to be good – the shell they create has a length of 1 atom or: d= 10^{-10}m and,

$$S_1=(10^{-10})^{21010}\ m^2=10^{-10}\ m^2 \qquad (1)$$

Therefore the ratio of charged to uncharged particles within this surface is

$$\frac{S_1}{S}=\left(\frac{n_1}{n}\right)^2=10^{-9}\Rightarrow\frac{n_1}{n}=10^{-3} \qquad (2)$$

In equation (2) we have taken for a typical surface of the rod S=10^{-1}m^2. If we apply Gauss's law to find the field created inside the material and just outside:

$$E=\frac{\sigma}{2\varepsilon\varepsilon_0}\quad (3)$$
$$\sigma=10^{-19}\,Cb/\lfloor 10^3(10^{-10})^2\rfloor m^2 \qquad (3)$$
$$E\approx 10^8\,V/m\quad (5)$$

In writing down equation (3) we have taken into account equations (1) and (2). The pressure being exerted is,

$$P=\sigma E\approx 10^6\,N/m^2\approx 10At\quad (6)$$

We deduce from these that the pressure is of the order of the atmospheric pressure but the extraction of a charge from the surface is also related to the surface cohesion forces. Especially in insulators it would seem that this is more related to the bulk modulus B~ 10 GPa rather than the cut off potential found in metals and in nth photoelectric effect.

The voltage required for break down in air is E_{break}~ 10^6 V/m. We observe this to be 1 order of magnitude less than is in our case, so, very close to the surface there is a continuous leakage of charge, that is, creation and annihilation of charges.

Second Case: Vaporization of water

For diatomic molecules such as is Oxygen or Nitrogen contained in the normal atmosphere, the heat capacity C~1Joule/gr =10^3 J/kg. We are going to make the assumption which will be justified later on from the numbers found, that the Bernoulli law holds:

$$P_{vapor}+\frac{1}{2}dv_{vapor}{}^{2}=P_{water}+\frac{1}{2}dv_{water}{}^{2} \tag{1}$$

$$P_{v}\approx 10^{5}\,Pa\approx 1At,\;\; d_{v}\approx 1kg/m^{3} \tag{2a}$$

$$P_{w}\approx 10^{9}\,Pa \quad d_{w}\approx 10^{3}kg/m^{3} \tag{2b}$$

It should become obvious right away that the density of vapor is similar to the density of any gas under normal conditions. Next in replacing the values given in (2b) and (2c) into equation (1) we find:

$$\begin{aligned}&10^{9}\,Pa+\frac{1}{2}\cdot 1kg/m^{3}v_{v}{}^{2}=10^{5}\,Pa+\frac{1}{2}10^{3}kg/m^{3}v_{w}{}^{2}\Rightarrow\\&\Rightarrow 10^{9}\,Pa+\frac{1}{2}\cdot 1kg/m^{3}v_{v}{}^{2}\approx\frac{1}{2}10^{3}kg/m^{3}v_{w}{}^{2}\end{aligned} \tag{3}$$

Now, the important thing about this trick is that if we were to use the kinetic theory of gases we would have found:

$$P_{o_2}\approx\frac{1}{3}d\left\langle v_{o_2}{}^{2}\right\rangle\Rightarrow\sqrt{\left\langle v_{o_2}{}^{2}\right\rangle}\approx 10^{3}m/\sec \tag{4}$$

In estimating the quantities involved in equation (3) we will rather equate the two kinetic energies assuming they are both of the order of magnitude of the bulk pressure of water and this brings forth the formation of a shell of equilibrium.

On equating so the two sides we get:

$$\frac{1}{2}\cdot 1kg/m^{3}v_{v}{}^{2}=\frac{1}{2}10^{3}kg/m^{3}v_{w}{}^{2}\Rightarrow v_{vapor}{}^{2}=10^{3}v_{water}{}^{2} \tag{5}$$

We may now recall that the thermal charge absorbed during vaporization – in which procedure the Temperature remains constant is about ten thousand times greater than the charge needed to elevate the temperature by one degree. This is also the order of magnitude of change of kinetic energies as seen from equation (5). It is only natural that a shell or surface layer would occupy a region where there may be some gradients of the thermodynamic quantities like $\Delta T \sim 3^0C$ even if the theory claims temperature remains constant. What is deduced from here is also that the drop of Pressure is not extreme within that surface layer and it should not be and this is also related to the lever rule in that in calculating the thermodynamic quantities such as is Pressure of steam we take in mind the density and reach out for the two extremes in the P-V diagram of water- vapor.

And now a question arises indeed: Why are the partial pressures of water vapor and air so different? It depends first of all on what is regarded as pressure of steam: It is also the very pressure of the walls of bubbles that must be enormous compared to the atmospheric one that must also be born in mind. Are the pressure we are referring to the pressure of water droplets of steam formed and the pressure coming from kinetic effects a different thing? May we actually even talk of droplets? If we do we find out in what follows that their curvature is associated with the kinetic pressure from Lavoisier's law. First of all one might say it is just a matter of big cross section and that there are only H_2O molecules and that the intermolecular forces make the difference. If one looks up at the literature then he may find that the same model of steam is applied to second order transitions like superconductivity. What also needs mention is that in semiconductor physics and other areas of solid-state physics the talk is on holes and electrons. Scientists use this model all over and we may recall a name found for the liquid drop model of the nucleus. However in classical physics at least the model suffers slightly in that we are talking about a surface having curvature approximated by the second derivative of a parameter h used to define it, just like in quantum theory the values of the wave function psi (ψ) may also be used to label surfaces. However we may not absolutely answer the very simple question whether we are talking of droplets of specific volume. The talk is general about a surface used as order parameter.

The author's personal view on the matter is that in classical physics we always have in one form or the other some boundary conditions defining the system. Some very good arguments for that are that vapor condenses on surfaces or that we all know that imperfections favor condensations and the bubble chamber where the course of ionized atoms may be watched. It is because molecules need a lot of speed to escape the surface let us calls it "potential" that this mixing of the concepts gas-liquid occurs. The starting velocities of escaping water molecules let us say inside the liquid are similar to those of air molecules outside while outside in the atmosphere the oxygen and nitrogen molecules inherit the cohesion forces of water perhaps in a poetic sense. But as can be seen from other examples we always need the boundary conditions.

Third Case: Brownian movement

Brownian movement was first observed by Albert Einstein (Einstein, 1956). In that case one may not actually speak of velocity. The movement is truly rare. One might actually think that if and when possible the let us call it so "arachnidan – like movement" could actually unfold under the influence of some kind of a field. This is usual kind of talk in statistical physics (Kubo, 1965). The actual path walked should rather be named as $<\Delta x^2>$ instead of $<x^2>$ to avoid some confusions. So, $\Delta x^2=2Dt=K_BT/6\pi\eta r$ with r measuring the radius of the pollen ball according to theory where D is the diffusion coefficient. The very fact alone, that the diffusion coefficient comes in, justifies the following talk.

As an intervention we need to mention that if one wants to find out more one might refer to a description of white noise in electronics (Dunlop and Smith, 1995).

Now, since we may not exactly talk of a velocity through taking the square root of length deviations, we may simply refer to something else: Δx^2 does have dimensions of surface. Brownian movement was actually observed on pollen particles swimming on the surface of water, so surface tension is definitely an influence there and the viscosity coefficient η appears naturally but it refers rather to the layers separation effect.

Fourth Case: Passage of Electric Current through a Conductor

In the case of electric current passage through a conductor, the electrons kind of form surface layers together with the ions which form the other capacitor layer and thus so create capacitance effects. When one electron should depart from these layers the static electrodynamics effects cease for the following reason: The conductor does have a crystal-like structure. The inner electrons of the atoms that make it do have such a structure indeed when seen as a whole. Now, in the case when one of the free electrons departs from its capacitance –like surface it reacts with one of the inner electrons in the equivalent atoms surface. But then that atom surface layer opens up suddenly and shielding stops thus there is emission of photon (rather it may be phonon but this is not the point here of such a deep discussion). So, in that case of interaction with the atom ions there is actual current within the conductor perpendicular cut, but otherwise the very movement is that of an electron gas – rare. Indeed in the z-axis the electric field propagates the signal rather – it is not an actual movement of electrons because the velocities are relativistic, as we all know of.

Fifth Case: A Study of Electron Shells in the Hydrogen Atom With Simple Methods

In the following we are going to discuss a very special property of Schrodinger's equation. It is the fact that once we find some quantized values of the operators, from Sommerfeld's rules of quantized angular momentum or by using the official results of mean values of the operators obtained from detailed solution of the aforementioned equation we may input them in Schrodinger 's equation it remains unchanged. The connection with the material dealt with in the paper comes into light as we associate with all that rescaling the Hydrogen polarized electron shells, as we would like to name them.

We are going to use Schrodinger's equation in its radial form for clarity:

$$\begin{gathered}\frac{d^2[r\psi(r)]}{dr^2} = \hat{p}_r^{\,2}\left(\hat{r}\psi(r)\right) = \left[\hat{E} - \hat{U}(r) - \frac{\hat{L}^2}{2m\hat{r}^2}\right]\hat{r}\psi(r) = \\ \left[i\hbar\frac{\partial}{\partial\hat{t}} - \hat{U}(r) - \frac{\hat{L}^2}{2m\hat{r}^2}\right]\hat{r}\psi(r)\end{gathered} \tag{1}$$

In equation (1) we shall use the following transformations naturally derived from Sommerfeld orbit quantization:

$$\begin{gathered}L = mvr = n\hbar \\ \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0 r^2}\end{gathered} \tag{2}$$

This is the result of the so-called rescaling:

$$\begin{aligned}
&\hat{r} \rightarrow n^2 \hat{r} \\
&\hat{L} \rightarrow n\hat{L} \\
&\hat{p} \rightarrow \frac{1}{n}\hat{p} \\
&\hat{U}(r) \rightarrow \frac{1}{n^2}\hat{U}(r)
\end{aligned} \tag{3}$$

We will come to see that this change cast upon the operators will bring us to the new wave function. The first term of equation (1) is transformed to:

$$\frac{d^2\lfloor n^2 r\psi(r)\rfloor}{(dn^2 r)^2} = \frac{d}{n^2 dr}\left(n^2\psi(r) + n^2 r\frac{d\psi(r)}{dr}\right) = 2\frac{d\psi(r)}{dr} + \frac{d^2\psi(r)}{dr^2} \tag{4}$$

The angular momentum term is transformed according to equation (2) simply. The result is:

$$\begin{aligned}
&2\frac{d\psi}{dr} + \frac{d^2\psi}{dr^2} = E_0 n^2 \hat{r}\psi(r) - \left(\frac{U(r)}{n^2} + \frac{\hat{L}^2 n^2}{2mn^4 r^2}\right) n^2 \hat{r}\psi(r) \Rightarrow \\
&\Rightarrow \frac{d^2[r\psi(r)]}{dr^2} = \left(\frac{E_0}{n^2} - \frac{U(r)}{n^2} - \frac{L^2}{2mr^2 n^2}\right) n^2 \hat{r}\psi(r) \Rightarrow \\
&\Rightarrow \frac{d^2[r\psi_n(r)]}{dr^2} = \left(E_n - U(r) - \frac{L^2}{2mr^2}\right)\hat{r}\psi_n(r)
\end{aligned} \tag{5}$$

So we have the original equation back except that the transformation of the spatial operators brought about the new wave function ψ_n together with what is its identity E_n. This is a situation that resembles a technique used in labeling the volume swept in liquids arising from the need to follow the particles in their flow (Salmon, 1998).

$$\begin{aligned}
&\psi \rightarrow \psi_n \\
&E_0 \rightarrow E_n
\end{aligned}$$

Yet what happens with the volume is what will be discussed from her and now on. According to the virial theorem (Park, 2005)

$$\frac{1}{2}\left\langle \vec{r}\cdot\vec{F}\right\rangle = -\frac{1}{2}\iiint|\psi|^2 \vec{r}\cdot\nabla U dV = \frac{1}{2}\int \vec{P}\cdot\vec{E} dV = -\left\langle K.E.\right\rangle \tag{6}$$

Equation (6) takes the form:

$$\begin{aligned}
&\left\langle \vec{r}\cdot\vec{F}\right\rangle = -\iiint|\psi|^2 \vec{r}\cdot\nabla U dV = \int \vec{P}\cdot\vec{E} dV = -2\left\langle K.E.\right\rangle \Rightarrow \\
&\Rightarrow \left\langle Kinetic\ Energy\right\rangle = \frac{u}{2}\left\langle Potenlian\ Energy\right\rangle = -\frac{1}{2}\left\langle U\right\rangle \Rightarrow E = \frac{1}{2}\left\langle U\right\rangle
\end{aligned} \tag{7}$$

Equation (7) is valid for potentials of homogeneity degree u=-1 as is the Coulomb potential. From a look at equation (6) we see that Energy is defined by the work of Polarization that changes with a change in the wave function. It is those shells in Hydrogen atom, which carry the energy in a sense because they are polarized. Now what has to be noted in accordance with what was previously discussed is that the operator of Force acts on the potential alone as is stated in standard quantum mechanics textbooks and not on ψ. This fact brings allowance into using the methods of rescaling resulting in transformations of Schrodinger equation as before. What has changed now is the Normalization factor 1/N which needs to be carried along for the energetic considerations to be valid and would not alter anything in Schrodinger equation.

Discussion

The mention of the order of magnitudes involved in case 1 may be partly extracted from the first – year physics textbook of reference (Ohanian, 1989). The practical use of this study is what may be called a common practical knowledge that when the atmosphere is wet electrostriction experiments are hard to accomplish.

Concerning the second case studied the reason we used Bernoulli law is that under usual thermodynamic considerations used by equation (4) the density is assumed that of droplets, thus the water density and we have big density, smaller velocity and the kinetic energy of air is absorbed. But in this case we are talking about a stream of particles and this is what makes the difference.

The reasoning behind all this is described in what follows: You may imagine tubes of particle flux. Since the gas is not dense, some assumptions are not satisfied. For a) The number of molecules N is not big enough, b) The microscopic balls which for the gas are not small compared to macroscopic dimensions, c) Brownian motions should have an effect through continuous hitting of say the liquid molecules to the bubbles inside the water can or hitting of the gas molecules on to the steam particles above the surface of the liquid. The result of all this is that there exists an upward motion with a velocity, which we will calculate in what, is to follow plus a mean velocity. This is called partial equilibrium. One might imagine that the kinetic energy of the steam gas has an influence on the magnitude of the steam particles.

The connection with Brownian movement is not bad for conceptual reasons. For this case one might indeed refer to case 1 for deeper connection. There is also a connection with shells in the hydrogen atom but it takes a very deeper talk to bring such a thing into surface. For more information one may look up at the book (Ashcroft and Mermin, 1976) on solid-state physics*. The conceptual connection is brought to mind as to relevance with the Free energy having a term with a surface tension coefficient. Experimentally such situations may be observed say in the X-ray or cathode – ray tubes sometimes as surfaces moving (Darigol, 2000)

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