Experiment with Conical Pendulum

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Abstract

Conical pendulum is similar to simple pendulum with the difference that the bob, instead of moving back and forth, swings around in a horizontal circle.Thus, in a conical pendulum the bob moves at a constant speed in a circle with the string tracing out a cone .This paper describes an experiment with conical pendulum, with determination of g from the dynamics of the pendulum bob.. The fact that, with increasing speed of revolution, the horizontal plane of rotation shifts towards the point of suspension is demonstrated with the governing equation $\omega^2 h$ = *constant* = *g. It is also shown that, in this case, the tension on the string approaches the centripetal force on the bob. Possible demonstrations like revolving planet with spin motion and vertical pendulum are discussed.*

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INTRODUCTION

Pendulums in physics are very basic and are of historic importance. ("Pendulus" means "hanging") .Galileo (around 1602) studied pendulum properties after watching a swinging lamp in the cathedral of Pisa's domed ceiling. Robert Hooke (around 1666) studied the conical pendulum and was the first one to design simple experiments which he presented in the Royal Academy, in order to understand the planetary orbits of the solar system [1, 2, 3.] Through centuries, conical pendulum is attracting scientists, teachers and finding its applications in different disciplines modern science. Few recent examples are Chaos in Robert Hooke's Inverted Cone [4], Robert Hooke's Conical Pendulum from the modern viewpoint of Amplitude Equations and its Optical Analogues [5].

Just as simple harmonic motion can be best understood with simple pendulum, the uniform circular motion can be demonstrated with conical pendulum. Conical pendulum is an extension of simple pendulum in which the bob, instead of moving back and forth, moves at a constant speed in a circle in a horizontal plane. Thus together with the string the bob traces out a cone. Spherical pendulum and vertical pendulum are the special cases of conical pendulum. In the spherical pendulum the bob traces an ellipse where as in the vertical pendulum object is free to execute a vertical circle about the point of suspension. Conical pendulum illustrates uniform circular motion, and the other cases are representative of a non uniform circular motion.

Although demonstrations of conical pendulum are much easier, actual experiments yielding correct results are not trivial. This is because the precise measurements of the angle of the cone or the height of revolving plane from the point of suspension are difficult.

THEORY

As shown in the figure 1. A bob of mass m is attached to the end of a light inextensible string of length ℓ whose other end is attached to a rigid support. The bob moves with angular velocity ω such that it executes a horizontal circular orbit of radius r. Let h be the vertical distance between the support and the plane of the circular orbit and θ be the angle subtended by the string with the downward vertical.

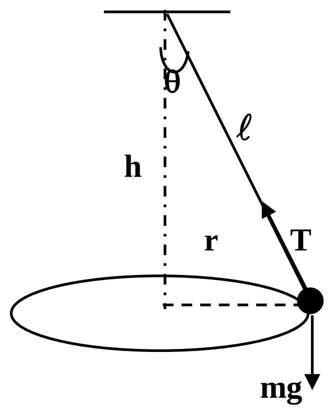

Fig.1. Conical Pendulum

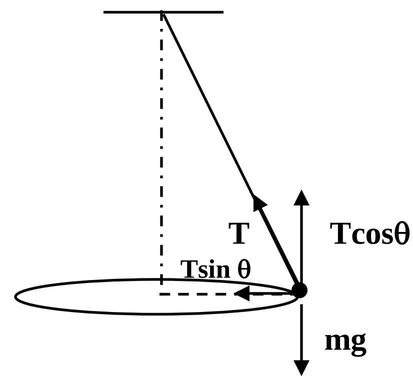

Fig. 2. Forces Resolved

As shown in Figure 1, the two external forces are acting on the bob of conical pendulum

1. The tension T in the string which is exerted along the line of the string acting towards the point of suspension
2. The weight of the bob mg acting vertically downwards.

Tension T on the string can be resolved into vertical and horizontal components .As seen in figure 2, the component $T\cos\theta$ acts vertically upwards and the component $T\sin\theta$ acts towards the center of the circle.

Force balance in the vertical direction yields

$$T\cos\theta = mg \qquad \ldots \quad .. \qquad ..(1)$$

In other words, the vertical component of the tension force balances the weight of the object.In a horizontal direction the system is imbalanced. The horizontal component of the tension in the string gives the bob acceleration towards the centre of the circle (Centripetal force).

Thus,

$T \sin\theta = m v^2 / r = m \omega^2 r$ ... . (2)

Taking ratio of equations 2 & 1 we get

$\tan\theta = \omega^2 r / g$ ... . (3)

By simple geometry of Figure 1, $\tan\theta = r / h$, substitution in equation (3) gives

$\omega^2 h = g =$ constant .. .. .. (4)

Thus as ω , the angular frequency of the revolving bob increases, the projection of pendulum length on the vertical axis decreases to keep the product $\omega^2 h$ constant. That is the revolving horizontal plane of the bob gets lifted towards the point of suspension. This fact is demonstrated during experiment. Moreover the constant gives the value is of gravitational acceleration g.

EXPERIMENTAL

Experimental procedure lies in measuring the periodic time T′ of revolution and the vertical height h . The angular frequency ω is simply $2\pi/T'$. However generally, it is difficult to measure height h of the rotating pendulum with precision.

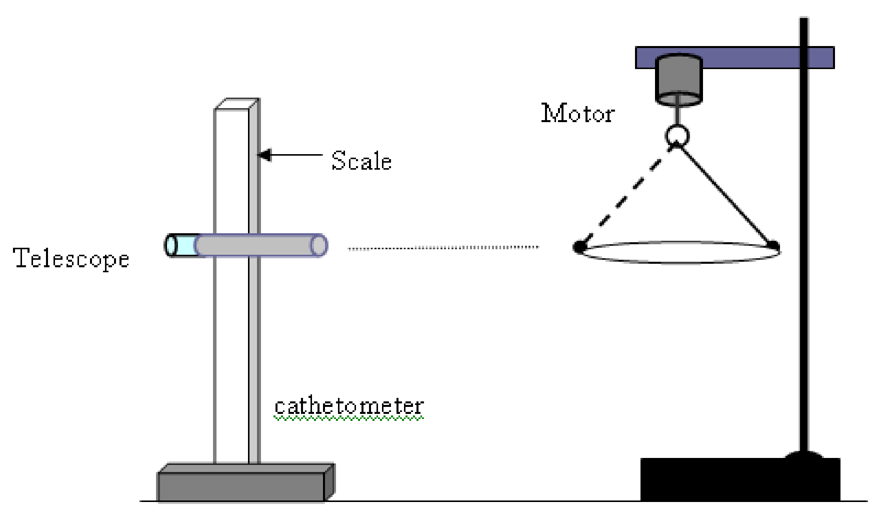

Figure 3 (a)

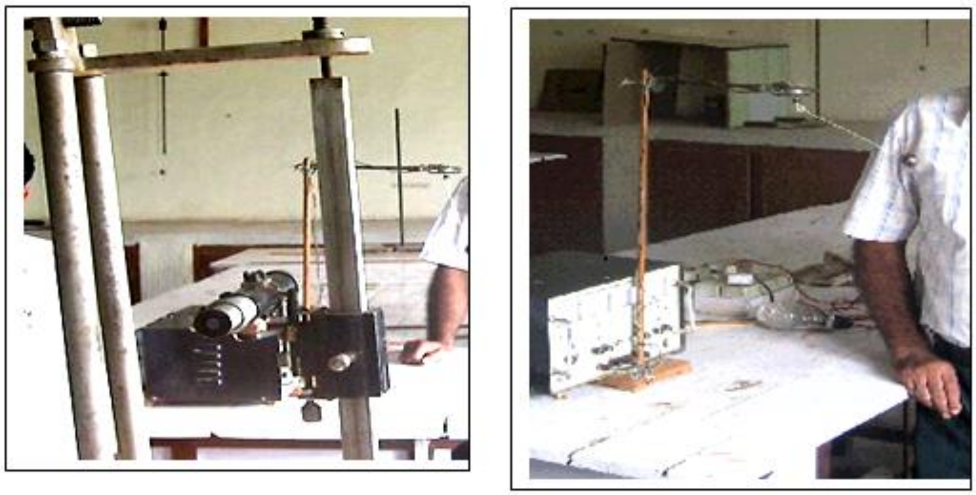

Figure 3(b) Photographs of the Experimental Setup

Figure 3(a) shows our experimental arrangement. Photographs of the setup are given in Figure 3(b) . A pendulum of length ℓ is attached to the shaft of a small 6 V DC motor whose rotational speed can be varied by changing the voltage. The length ℓ and the speed of motor are so adjusted that the period for 20 revolutions is measurable with a necked eye , at least for four to five values of angles of rotation . Focusing the telescope of the cathetometer on the revolving pendulum and noting down the corresponding reading on the scale with respect to the point of suspension , measures the height h of the rotating pendulum. With increasing speed of revolution, the plane of revolution it becomes slightly difficult to trace out the pendulum trajectory in the telescope. The observations are tabulated as in Table I.

OBSERVATION & RESULTS

Mass of pendulum bob m = 72 gm
Length of pendulum ℓ = 19.90 cm

Table 1: Experimental Observations

Obs. No.	Height (cm)	Time for 20rev. (sec)	Periodic Time T′(sec)	Angular Frequency $\omega = 2\pi/T'$ (radian /sec)	ω^2	$g = \omega^2 h$ cm/s^2
1	18.7	17.0	0.85	7.392	54.65	1021
2	15.9	15.5	0.775	8.108	65.74	1045
3	9.75	12.5	0.625	10.05	101.09	985
4	5.1	9.2	0.46	13.66	186.63	951
5	4.0	8.0	0.4	15.71	246.80	987

We see that the product $\omega^2 h$ is fairly constant within the range of experimental errors. Its average value is 997 cm/s^2 which are close to the standard value of gravitational acceleration 980 cm/s^2. Improving on experimental measurements, a better agreement can still be arrived.

Now we proceed to compute physical quantities related to circular motion of the bob ie. centripetal force and tension on the string. For this we use eq.(2) and do little substitution in terms of measured parameters h and ℓ .Thus,

$T \sin\theta = m v^2 / r = m \omega^2 r$ .. .... (2)

Tension $T = m \omega^2 (r / \sin\theta)$ and from figure 1. , $r = \ell \sin\theta$,we get

Tension $T = m \omega^2 \ell$ .. … (5)

Also from figure 1, $\cos\theta = h/\ell$,with $\sin^2\theta + \cos^2\theta = 1$, we get $\sin\theta$ as $\sin\theta = \{\mathrm{sqrt}(\ell^2 - h^2)\} / \ell$ and the centripetal force becomes

Centripetal Force = $T \sin\theta = m \omega^2 \{\mathrm{sqrt}(\ell^2 - h^2)\}$ .. . .. (6).

Table II summarizes these results.

Table II : Parameters related to Circular Motion

Obs. No.	Height h cm	Angular Frequency ω rad /s	Tension T on string dyne	Centripetal Force dyne
1	18.7	7.392	7.83×10^4	2.67×10^4
2	15.9	8.108	9.36×10^4	5.66×10^4
3	9.75	10.05	14.4×10^4	12.62×10^4
4	5.1	13.66	26.5×10^4	25.80×10^4
5	4.0	15.71	35.1×10^4	34.58×10^4

The variation of tension T and centripetal force with the angular frequency of revolution is plotted in figure 4.

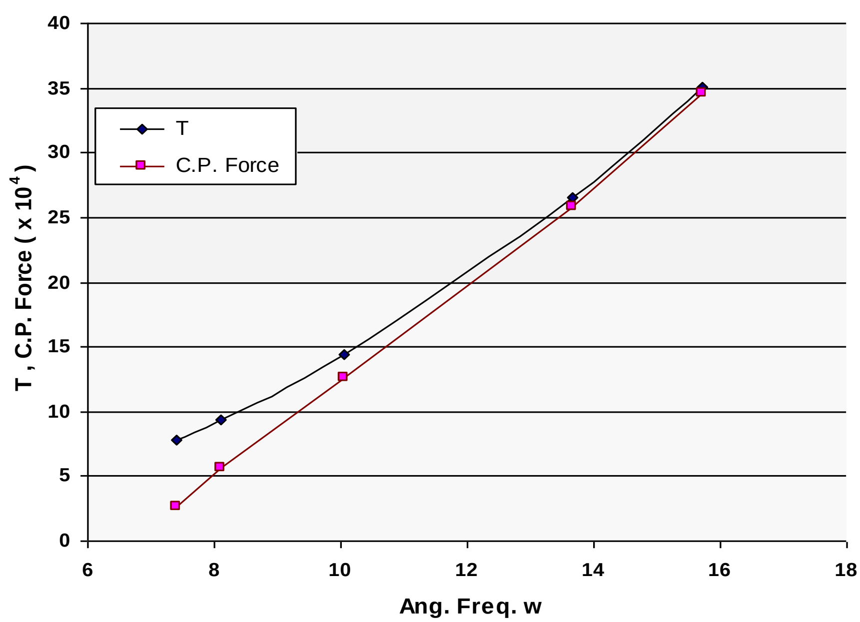

Figure 4 : Variation of Tension T and C.P. Force with Angular Frequency

We see that initially when the bob rotates slowly at lower frequencies, the tension on the string is larger than the centripetal force on the bob. As the speed of rotation increases both of them increase in the same fashion and finally they attain almost the same value. In the actual experiment , the revolving horizontal plane of the bob gets lifted towards the point of suspension and the conical trajectory of the string becomes a circular one. For circular motion tension T and centripetal force are the same.

CONCLUSION

In conclusion the Conical Pendulum is well illustrative of uniform circular motion and determination of gravitational acceleration g is possible with simple arrangements. We are trying other possibilities to improve on measurements of h and period of oscillation. Still, the present measurements are quite accurate as the maximum percentage error ranges from +6.63 % to – 2.9 % .

FURTHER DEMONSTRATIONS

1. Centrifugal Reaction

When we replace the bob with a small plastic cylindrical container filled with water, while revolving the water does not come out even when it is horizontal to table at highest speed of revolution. This demonstrates the effect of centrifugal reaction on the revolving water.

2. Vertical Pendulum

In vertical pendulum the motor is clamped horizontal to table so that the bob describes a circle in a vertical plane. Photograph of the vertical pendulum is shown in Figure 5(a) and the forces acting on it at different radial positions are shown in given in Figure 5(b)

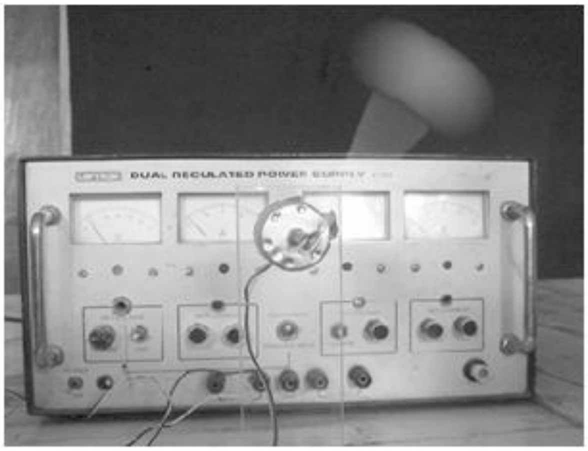

Figure 5 (a) :The Vertical Pendulum

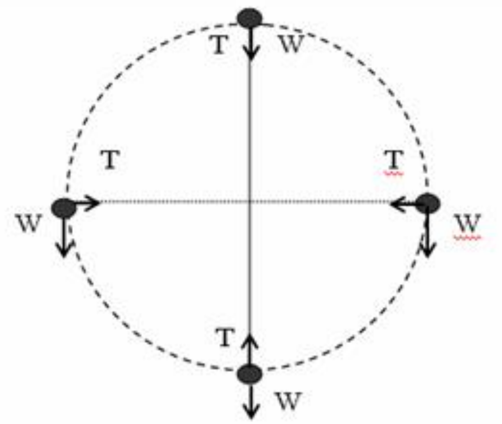

Figure 5 (b) : Forces on bob at different radial positions

It is shown by Richard Fitzpatrick [6] that the condition for the object to execute a complete vertical circle without the string becoming slack is

$$\omega^2 r > 5g$$

If the object is attached to the end of a rigid rod, instead of a piece of string, the condition is

$$\omega^2 r > 4g$$

The motion is much easier when a solid rod instead of a string is used.

This is simply because a solid rod can bare a negative tension when the bob is at the top position above the pivot point ,rather than a string. The rigidity of the rod helps to support the object at this position. In the demonstration we note that the motor draws smaller current with the rod in stead of a string for the bob execute a vertical circle of same radius r.

3. Planetary Motion

In conical pendulum when the bob is replaced by another small motor to which a ball is attached, the system is reduced to a one in which the ball while spinning about its own axis revolves simultaneously. Figure 6 illustrates this concept. With an elliptical path it represents a motion of a planet. We are trying for this demonstration.

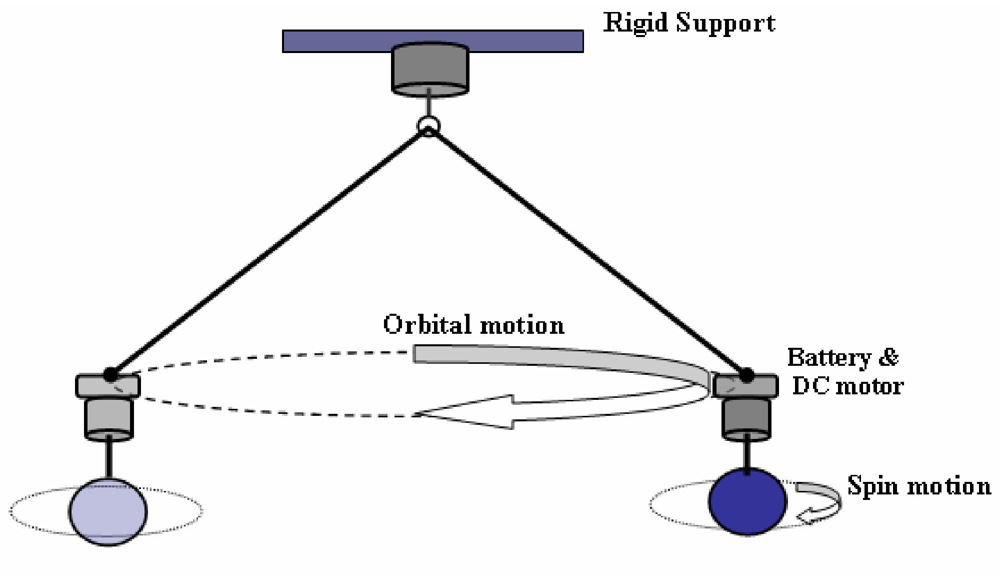

Figure 6: Demonstration of Planetary Motion.

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