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Integrating Writing into Mathematics Classroom to Increase Students' Problem Solving Skills

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ABSTRACT

Middle school students lack efficient problem solving skills due to several factors. The writing process has been linked to the development of students' problem solving skills. The present study investigated the impact of the writing process on the mathematical problem solving skills for middle grades students. N = 96 students participated in a six weeks, after school STEM program, and they were randomly assigned into two groups: one focused on the writing process with mathematical problem solving and the other on homework/high stakes test preparation with mathematical problem solving. In this quantitative study, the results provide evidence that the students from the writing process group were more likely to generate and apply better problem solving skills as compared to the control group. This study further contributes to the support and importance of integrating different subjects across the generalized learning realm.

Keywords:
Problem solving, writing process, metacognition

Introduction

Many middle grade students lack efficient problem solving skills. To increase students' problem solving skills, their mathematical reasoning first needs to be developed by fostering their mathematical thinking. Higher levels of cognitive thinking are essential for advancing problem solving skills, which serves as a key element for students in mathematics classrooms today. Seto and Meel (2000) noted that one of the crucial changes in mathematics teaching and learning is considered as integrating writing into mathematics classrooms. This type of integration increases students' mathematical problem solving skills because writing in mathematics requires students to demonstrate how and why they know things, as well as what they already know (Banger-Drowns, Hurley, & Wilkinson, 2004). However, there are no studies focusing on writing as a mediator for story problem solving or story problem posing. The purpose of the present study was to reveal the role of the writing process as a mediator of students' problem solving skills.

Perspectives or theoretical framework

The "problems" students solve are the most basic unit of classroom instruction (Arbaugh & Brown, 2004). Critical thinking, higher order reasoning, and creativity required to find a solution determine the quality of a mathematical task (Polya, 1967; Schoenfeld, 1985; 1987; Selden, Selden, Hauk, & Mason, 2000). Correlations between increased student achievement and higher quality questioning facilitated by the solving of complex problems have been documented (Lampert & Cobb, 2003). In short, questions control

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student learning (Manouchehri & Lapp, 2003) heightened expectations through challenging questions yielded heightened achievement (Piccolo, Carter, Harbaugh, Capraro, & Capraro, 2008).

Difficulty versus Complexity and Scales of Cognitive Complexity

The quintessential quality of a mathematical task is its cognitive complexity (Webb, 1997), as opposed to its difficulty. Even though "complexity" and "difficulty" are often used interchangeably, there is a big difference between the two terms. If a problem is difficult, then the solution requires much effort. The extended duration of a solution (e.g. moving 500 boxes from one room to another) makes a task difficult not complex. There exists no question about how to solve a difficult task. If a problem is complex, then the solution is complicated in structure. Alternatively from the previous example, there is no immediate procedure available to solve a complex task (e.g. arranging 500 boxes to fit into a limited space). The greater the recognition of the difference between the two terms, the greater the insight on how to classify students' abilities and their level of cognitive thinking.

Levels of cognitive capability have been broken down into various models and scales to assist in categorizing the different ability levels of problem solving skills that require higher mathematical thinking. The Van Hiele model was developed to improve teaching by considering students' thought processes (Pegg & Davey, 1998). This model, which includes five levels – visualize, analyze, generalize, deduce, and rigor, was developed for geometry branch of mathematics (Van Hiele, 1973). The first level is basic recognition of shapes such as a circle is a circle and a square is a square. At the second level a student begins to analyze the shapes such as a square has 4 equal sides and angles. At the third level, students begin to understand relationships among different shapes such as a square is always a rectangle, but a rectangle is not always a square. At the fourth level the student should understand geometric proofs and theorems. The fifth level is the highest level of cognitive demand from the student. The student understands that nothing is concrete and he/she must think in a more abstract manner.

Another commonly used tool to identify and measure cognitive capability is Webb's (1997) Depth of Knowledge Scale with four cognitive levels: recall, application, strategic thinking, and extended thinking. Level one is simply recalling basic facts and definitions and includes terms such as "identify" or "measure". Level two requires students to develop a game plan on how to approach a problem that requires a slightly higher level of thinking than the previous level and includes terms such as "classify" and "estimate". Level three is where the cognitive demands begin. It requires students to think in a more complex and abstract manner and also for students to explain the reasoning behind their problem solving approach. Level four, also known as extended thinking, requires a much more extended period of time. Students must "make several connections relate ideas within the content area or among content areas and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level" (Webb, 1997, p. 23). To achieve the highest cognitive capacity, writing can be a helpful practice for learner; thus teachers in certain fields including but not limited to mathematics and geometry can integrate writing into their classrooms to increase their students' cognitive level, and enable them to achieve at the highest level.

Van Hiele's geometrical understanding model follows the pattern that as the levels of the model/scale increases the amount of cognitive thinking that is required from the student increases as well. The higher the level of cognitive thinking the higher level of mathematical reasoning skills students can attain. In order students to achieve the highest level of Van Hiele's scale, they need to understand geometric proofs and theorems. However, Battista and Clements (1995) noted that mathematical (algebraic) reasoning needs to be developed before students can be expected to understand geometric proofs and theorems; thus, to achieve the highest level of Van Hiele's geometrical understanding model, students first need to develop mathematical (algebraic) reasoning. Higher levels of cognitive thinking are essential for advance problem solving skills, which serve as a key element for students in mathematics classrooms today.

Writing to increase students' problem solving skills, and cognition in mathematics. National Council of Teacher of Mathematics (NCTM) has stated that all school-aged students in mathematics classrooms should not only be able to communicate by using mathematical language, but also be able to share explicitly their mathematical thinking with others (NCTM, 2000). Adams (2010) has noted the importance of communication in mathematics by saying there would be no mathematics without a language. While mathematics has had itself a language for communication (Capraro, Capraro, & Rupley, 2011), there are some other beneficial ways to assist communicating in mathematics. Using writing for this purpose in mathematics classroom has received increased attention (Meiner & Rishel, 1998). One research project emphasized the importance of writing as one of the crucial changes over the past couple decades in mathematics teaching and learning by using it as a tool in the mathematics classroom (Seto & Meel, 2006). Research in the mathematics education era has showed that integrating writing into mathematics classrooms increases both students' mathematical content learning (Meel, 1999), and students' mathematical problem solving skills (Bagley & Gallenberger, 1992). Later, NCTM (2000) had also noted that integrating writing into mathematics classrooms develops students' mathematical thinking abilities and skills because writing in mathematics requires students to demonstrate how and why they know things, as well as what they already know (Banger-Drowns, Hurley, & Wilkinson, 2004). There are various ways to integrate writing into mathematics classroom. One of the most common is to keep a journal in mathematics classrooms. The selection of which writing methods used should not be an arbitrary method; instead, teachers need to select the most appropriate method according to their specific purposes (Burton & Morgan, 2000).

Keeping a journal in mathematics classroom. Keeping a journal in mathematics classrooms has demonstrated some benefits in terms of students' mathematics content learning, higher cognition, and problem solving skills. Watson (1980) stated that students who kept a journal achieved higher mathematics test scores than students who did not. Similarly, Borasi and Rose (1989) found writing a journal in the mathematics classroom not only increased students' mathematics content knowledge, but also increased students' problem solving abilities and skills. By keeping a journal, students had an opportunity to see how they do mathematics (Borasi & Rose, 1989). In turn, it has helped students construct new content knowledge (Hayes, 1996) in mathematics. Using writing to explain mathematical topics in a journal not only provides teachers with records of students, but also helps students clarify their own thoughts (Farris, 1993).

Through keeping a journal, students become more capable of examining mathematical thoughts and ideas (Linn, 1987; Rose, 1990), and they can construct knowledge in their own language (Goodkin, 1982). The mastery of content vocabulary and understanding mathematical concepts are directly related with each other (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008). In other words, students' vocabulary development is vital to increasing students' conceptual understanding in mathematics (Capraro & Joffrion, 2006). In order to avoid students' mathematical misunderstanding, students need to see the relationship between concepts and terms they already know while they learn new mathematical vocabulary (Ashlock, 2006). However, mathematical language students learn in schools mostly has not related with either their spoken language at home (Crillo, Bruna, & Herbal-Eisenmann, 2010) or oral language that they face in their daily-lives (Capraro, Capraro, & Rupley, 2011). Keeping a journal in mathematics classroom provides students an opportunity to share their ideas by using their own words. Mathematics teachers should keep in mind that students should not move too quickly to use precise mathematical terms. Steele (1999) noted that teachers should not be in a hurry for students to use precise mathematical terms without allowing them to explore, investigate, and explain ideas.

There are two strategies for keeping journals: 1) writing about what students have known or learned, and/or 2) writing about students' feelings about particular practices in mathematics (Ashlock, 2006). While the first method for keeping journal helps teachers see students' initial experiences about specific mathematical topics (Crespo, 2000), the second one, is a helpful practice for students to clarify their own thoughts (Farris, 1993). However, keeping a journal is not the only method to integrate writing into the mathematics classrooms. There are some other useful methods: 1) writing out definitions by using already known words, 2) writing test questions about what students learn, and 3) describing what is not understood (Ashlock, 2006).

Writing to understand students' mathematical background knowledge. Understanding students' thinking has been emphasized as one of the crucial parts of mathematics teaching and learning (NCTM, 1990). Unless teachers have asked for students' thinking, they cannot describe what students do and do not know (Ball, 1994). A learner-centered environment is one educational approach that can help teachers diagnose students 'previous knowledge, skills, attitudes, and beliefs (National Research Council, 2000).

When teachers use writing as a tool in the mathematics classroom, students take responsibility for their own learning (Mayer, Lester, & Pradl, 1983), and the mathematics classroom becomes a student-centered environment rather than a teacher-centered one. By asking students to write answers to critical questions, teachers can understand students' previous concepts, thus determining students' strengths and weaknesses (Bell & Purdy, 1985). Asking questions to determine students' prior knowledge is also known as "diagnostic teaching" enabling teachers to know the needs of each student (Ladson-Billings, 1995). Talking and writing mathematics are both useful for students to relate daily life to mathematics language. However, writing mathematics is more reflective in terms of students' understanding than talking mathematics (Ashlock, 2006). Both oral and written communication can also help teachers diagnose students' thinking and conceptual understandings. Integrating writing as a tool along with oral communication in mathematics classrooms offered some advantages for the teaching and learning process (Crespo, 2000).

One benefit of written communication is to provide effective feedback helping students to correct their misunderstandings and overcome their mathematics' difficulties. Because teachers need an extended amount of time for reading students' papers as opposed to listening to only a few students' oral explanations, it creates an enormous time constraint. However, this method affords the ability to carefully analyze each student's paper and provide feedback in an appropriate way (Crespo, 2000). "Rather than just scoring papers, we need to understand each student's paper diagnostically-looking for patterns, hypothesizing possible causes, and verifying our ideas" (Ashlock, 2006, p. 15).

Another benefit is to create a school community environment among teachers and students (National Research Council, 2000). One useful method to detect students' misconceptions is to create groups of teachers who analyze students' work together by sharing experiences (Ashlock, 2006). For example, when one teacher has difficulty understanding a students' paper, he or she receives an opportunity to ask other teachers. From this learning community, students receive quality feedback, and additionally the pedagogical knowledge of teachers is improved.

Writing process in mathematics classroom. The writing process provides enhanced cognitive opportunities for externalizing internal representations for direct interpretation; the writing process enables students to gather, analyze, and interpret data (Nahrgang & Petersen, 1986). Emig (1977) noted that writing helps students analyze, compare and contrast, and synthesize relevant information. In addition to these three benefits, writing enables students to organize their thoughts (Bagley & Gallenberger, 1992) by fostering the development of reasoning skills (Swafford & Bryan, 2000). The meaning of writing in order to learn mathematics is to understand, retain, analyze, and organize mathematical concepts (Flores & Brittain, 2003). The writing register and writing process provides a separate pathway for learning new content (Hayes, 1996) and has been shown to enhance the constructive process associated with cognition (Flower, Schriver, Carey, Haas, & Hayes, 1992). The writing process requires students to be aware of the subject (Odell, 1980). Recent research has suggested that students who used writing as a tool construct higher comprehension skills than students who did not use writing as a tool to learn (Duke & Pearson, 2002; Haneda & Wells, 2000). Integrating writing into mathematics classrooms has led to developing students' metacognition (Kreeft, 1984; Stanton, 1984) as students became aware of their own thinking (Bell & Bell, 1985). In turn, building metacognition through writing has led to an increased ability to problem solve. Because writing to learn has required self-questioning, using prior knowledge, inferring, and using imagination, students' thinking became critical, original, and insightful (Nagin, 2003). Through writing, students in mathematics classrooms were able to be aware of their understanding and express their confusions, beliefs, and feelings with others (Banger-Drowns, Hurley, & Wikinson, 2006). Specifically, the writing process has been linked to enhanced problem solving (Flower & Hayes, 1977). Writing in mathematics classes is the evidence of metacognition of students' problem solving process because it provides not only students with the realization of their own thinking about mathematics, but also enables teachers to see students' mathematical thinking (Pugalee, 2001).

Therefore, the combined theoretical framework of combining the structured writing process with mathematical problem solving was initially postulated and tested qualitatively showing that depth and structure of the mathematics learned was enhanced (Pugalee, 2001). In the present study, we use structured writing to quantitatively examine how the complexity, problem solving skills, and writing mechanics changed using two randomly assigned comparison groups.

A Statement of the Problem. Middle grades students lack fundamental problem solving skills due to many factors. In part, second language issues can manifest in poor story problem solving. There are no studies that focused on writing as a mediator for story problem solving or story problem posing. Problem solving is an integral part of most mathematics classrooms; however, research has shown that students have difficulty understanding, solving, and interpreting problems due to several factors one of which is second language acquisition. Integrating writing into mathematics classrooms is considered one of the crucial changes over the past couple decades in mathematics teaching and learning (Seto & Meel, 2006). Specifically, Flower and Hayes (1977) indicated that the writing process has been linked to the development of students' problem solving skills.

Methods and Procedures

The middle grades students were from one Texas Education Agency (TEA) designated Science, Technology, Engineering, Mathematics (STEM) academy during the summer of 2011. All 259 students were invited to participate; 45 students did not choose to participate, and of the remaining students, 118 parent permission forms were either not returned or did not give permission to participate in the study. Students ($N= 96$, $n_{female} = 51$) participated for six weeks of supplemental instruction during an afterschool program that met two days per week for 1 hour and 15 minutes. The teacher was trained by the research team to implement the instruction but was not a research team member. All materials were developed by the research team. Students were randomly assigned to one of two opportunities approved by Human Subjects. The first was the writing process, and the second was a homework/high stakes test preparation opportunity. The former focused on interpreting information, writing mechanics, generating story problems, and solving each other's generated problems. Students generated their own problem by rewriting each item into a new context that 1) either used the same numbers with a different context and operation or 2) new numbers and context, but the same operation. The latter focused on perfecting convergent answer skills by solving multiple choice test items typically missed in that school or providing homework. All participants were pre-post tested on writing skills and problem solving. The problem solving tests were comprised of three aspects: cognitive complexity, problem generation, and correct answers. Data were analyzed using Cohen's *d* effect size (Cf. Capraro, 2004; Capraro, 2009) for mean differences pre and post. Therefore, between group comparisons were not useful because the experiences were not similar. Growth from pre to post on the two instruments served as the salient metric. While some might be tempted to compare the post-tests means for each group, the research team felt the value added by each condition was the more important factor and simply determining one activity was better than the other to be less interesting (i.e., Zientek, Capraro, & Capraro, 2008).

Results

The purpose of this study was to reveal the role of the writing process as a mediator of students' problem solving skills. The results of the pretests showed that both groups performed at similar levels. Levene's test for equality of the variances was not statistically significant and the *t*-test of the means resulted in $p = .563$. Therefore, the assumption that the groups were similar due to the assignment process was verified based on pretest results. The pretest was not a vitally important component but necessary for ensuring similarity of the groups on the salient measure of the study. The validity risk of sensitivity to the test was therefore, warranted and the sensitivity could be reasonably assumed to effect both groups equally because they both had the same experience. The writing group had an overall growth in performance from pre to post of $d = 1.45$ composite across both cognitive complexity and problem generation. Correct answer growth was somewhat less dramatic $d = .384$. Results indicated the writing group outperformed the comparison group on the test of cognitive complexity (Cohen's $d = 1.34$) and problem generation (Cohen's $d = .888$). The problems generated by the writing group were 63% more likely to be solvable than the comparison group. The homework/high stakes group had more modest growth in cognitive complexity $d = .21$ and problem generation =. 51 but correct answer generation showed the greatest improvement in their correct answer responses than the writing group.

Discussion

According to the present study, the writing process helps develop students' problem solving skills. There are many reasons why integrating writing into mathematics classrooms increases students' problem solving skills. First, the writing process enables students to organize their thoughts (Bagley & Gallenberger, 1992). Some students are not flexible in organizing their thoughts about problems due to either the complexity or difficulty of the problems. However, making a plan through writing can assist students in overcoming the complexity of problems. For instance, teachers in mathematics classroom may require students to follow some strategies like Polya's 4 steps: understanding known and unknown, connection of known and unknown, making a plan, and checking and interpreting the results. Then, by following and writing down each step of some problem solving procedure like Polya's, students may overcome some of the complexities of problems. Second, the writing process enables students to gather, analyze, and interpret data (Nahrgang & Petersen, 1986). Some students have difficulty interpreting the solution of problems even when they are able to find the correct results. Not only students but also some teachers might have difficulty explaining the solution of problems due to the difficulty of problems. The reason why students have difficulty interpreting the problem might be due to students' lack of spatial thinking or mathematical imagination. However, through writing, students can explain their results to others. For instance, they can sketch with pictures, figures, or graphs to make the problems clearer and more doable. The writing process may enable students to solve and interpret problems because writing gives a visual image to an abstract problem that requires higher reasoning. Thus, the writing process may encourage students to solve difficult problems because writing makes difficult problems more concrete rather than an abstract or imaginary thing. Third, the writing process fosters the development of reasoning skills (Swafford & Bryan, 2000). Writing develops students' metacognition in mathematics (Kreeft, 1984; Stanton, 1984). Through writing, students clarify what and how they know some mathematical terms, topics, or axioms, so they became aware of their thinking. Integrating writing into mathematics classroom may also be a beneficial tool for teachers in understanding students' misconceptions and difficulties about certain mathematical topics because once teachers analyze each student's writing response carefully, they can see how students' ideas need be developed or changed. For instance, teachers want students to write answers to critical questions such as "What do you feel about inequalities?", "What so you know about equations?", "Do you like or hate polygons?", "What did you learn about functions?" Asking critical questions and expecting students to answer these in written format may help teacher diagnose students' misconceptions, beliefs, and understanding about certain mathematical topics. As a result, using writing as a tool in mathematics classroom not only increases students' metacognition but also helps teachers understand students' mathematical misconceptions, beliefs, understanding, and feelings.

The Educational or Scientific Importance of the Study

This intervention shows how important integrated knowledge is to generalize learning across disciplines. Using writing to help structure the mathematics learning resulted in greater than expected gains in problem solving.

Suggestions for Future Research

Research showed that integrating writing into mathematics classroom can substantially increase students' mathematics cognition, and the present study specifically noted that the writing process helped develop students' problem solving skills. However, the way teachers in mathematics classroom integrate writing is no more than paper and pencil procedure. Integrating writing into mathematics classroom via various social networking sites such as Facebook, Twitter, and Google+ may increase students' interest and motivation towards writing in mathematics classrooms.

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