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Evaluation of Number Sense on the Subject of Decimal Numbers of the Secondary Stage Students in Turkey

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ABSTRACT

The aim of this study is to examine number sense on the subject of decimal numbers of the secondary stage students in Turkey. Number sense, which can be described as well-understanding of numbers and calculations, is one of the main foci of mathematical education. When students improve their number sense, which are ability to estimate, mental computations and analytical thinking that will increase their use of mathematics in daily life. So as to measure students' number senses about decimal numbers, a test called Number Sense Test about Decimal Numbers which was prepared by the researchers was applied to 573 students (Secondary-stage students: 6th, 7th and 8th grades.) in 6 schools from the different regions in Turkey. Later on, in order to find out the possible solutions, 9 students were interviewed. As a result of the study, students' number senses on decimal numbers turn out to be very low. Root causes of these can be considered as the use of rule-based solution techniques, imperfect knowledge and misinformation about decimal numbers. This type of sense has a good correlation with students' ages, however, it turns out that there is no correlation between genders of the students and their number senses. In addition to these, moderate level correlation was found out between mathematical achievement and number sense on decimal numbers. It is believed that after the importance of number sense is acknowledged, the problems of failure will be prevented.

[1]

Keywords:
Number sense, decimal numbers, secondary stage students, Turkey

Introduction

The aim of the mathematical education in schools is that students can develop their mathematical thinking and use this ability in every part of their lives. Perry and Dockett (2002) suggested some powerful mathematical ideas, which young children can easily obtain. These mathematical ideas are mathematization, connections, argumentation, number sense and mental computation, algebraic reasoning, spatial and geometric thinking, data and probability sense. One of these ideas, number sense has been argued for 25 years (Dehaene, 1997; Greeno, 1991; Nickerson & Whitacre, 2010; Sowder & Schappelle, 1989) however with the help of new studies in Turkey, this concept will be much familiar (Harç, 2010; Kayhan Altay, 2010; Sulak, 2008; Yazgan, Bintaş, & Altun, 2002).

With the saying the least of it, number sense, which can be described as "a good intuition about numbers and their relationships" (Howden, 1989), is one of the best reflections of mathematics into daily life. It is because number sense refers to several important but elusive capabilities, including flexible mental computation, numerical estimation, and quantitative judgment (Greeno, 1991). These properties of number sense are the top-wanted outcomes of this mathematical education. Therefore developing number sense is

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the new focus of mathematics education. For example, by saying that number sense is an important subject, National Council of Teachers of Mathematics [NCTM] stated that math curriculum will help students understand numbers, different ways of representing numbers, relationships among numbers, and number systems; understand meanings of operations and how they relate to one another; compute fluently and make reasonable estimates (2000, p. 32).

In the literature, descriptions of "number sense" show the features of itself. One of the most common ones is the following: Number sense refers to a person's general understanding of numbers and operations and the ability to handle daily-life situations that include numbers. This ability is used to develop useful, flexible and efficient strategies for handling numerical problems (McIntosh, Reys, & Reys, 1992; Yang, 2005b). Estimation of the height of a desk that one student is going to use, estimation of the time that one will spend to go to one destination from another, predicting the sum of two -higher than 50- numbers should make higher number than 100, knowing that there are infinite numbers between 2.8 and 2.9 etc. can be given as a few examples of number sense.

The Components of Number Sense

Due to the similarities with "common sense" in terms of being difficult to notice and characterize, there are lots of classifications for the components of number sense (Reys, et al. 1999). One of the most basic classifications was made by McIntosh in 1992. In this classification, number sense includes 3 components: knowledge of numbers, knowledge of operations and applying knowledge of numbers and operations to computational settings.

In this research, based on the studies in the literature, four basic components of number sense is studied (Li & Yang, 2010; Markovits & Sowder, 1994; McIntosh, et al., 1992; NCTM, 2000; Yang & Wu, 2010). These are the followings:

a. Understanding the basic meaning of numbers: This implies to understand the base 10-number system (whole numbers, fractions, and decimals), including place value, pattern of numbers and using multiple ways to represent numbers (McIntosh, et al., 1992). Knowing that 10 times 10 or 2 times 50 makes 100, or 0.25 equals to 1 divided by 4 are some examples of this component.

b. Recognizing relative number size: This component determines the understanding of relativity of numbers and their absolute magnitudes. For example, ordering numbers correctly by ignoring the length of decimal number. Like 0.254 is less than 0.3.

c. Being able to use a benchmark appropriately: This implies that a person can use the benchmarks to solve problems flexibly and appropriately (McIntosh, et al., 1992). Such as a number like 1, ½ or 100 or personal attributes or encounters.

d. Being able to judge the reasoning of a computational result: This includes using mental computation strategies in order to solve some mathematical problems and to judge the reasoning of results. For example, dividing a number by ½ is the same as multiplying it by 2.

Literature

There are lots of international resources on number senses. In these studies, from different grades, students' number senses are examined in different countries (Alajmi & Reys, 2010; Howell & Kemp, 2006; Mohamed & Johnny, 2010; Yang, 2005b; Yang & Li, 2008; Yang, Reys, & Reys, 2009). Results showed that levels of students' number senses are not sufficient. In one of studies, 808 3rd grade students' number sense from Taiwan was examined (Yang & Li, 2008). Approximately, 34% of accuracy rate was obtained in each component of the number sense and this shows that students in Taiwan do not have good performance on number sense. Another study was carried also in Taiwan on 5th grade students to examine their success on number sense (Yang, Li, & Li, 2008). According to the study results, the students performed best on "recognizing relative number size" and performed worst on "judging the reasoning of estimates of computed results". Yang's (2005b) study showed that rather than number sense strategies, students tend to use rule based methods and written standard algorithm. Similar results were obtained on a study done with pre-service teachers (Yang, et al., 2009). According to the result of these studies, preservice teachers could not obtain good performance on number sense while they preferred to use rule-based strategies.

In addition to researches about students' number sense, in the literature, there also exist some researches about how different teaching methods affect students' number sense development (Griffin, 2004; Irwin & Britt, 2005; Kaminski, 2002; Markovits & Sowder, 1994; Yang, 2003, 2005a; Yang, Hsu, & Huang, 2004; Yang & Wu, 2010). Researchers showed that use of realistic number sense activities has a positive effect on number sense improvement.

In other studies on number sense, its relation with other abilities is examined (Louange & Bana, 2010; Pike & Forrester, 1997; Tsao, 2004; Yang & Huang, 2004). In these studies, it was observed that there is a correlation between number sense and estimation, mental computations and problem solving.

Number of studies on number sense in Turkey is limited with thesis. In the first study, 95 6^{th} grade students were tested. According to results of this study, only 9% of students used number sense (Harç, 2010). Another study was carried with secondary stage of primary school students so as to see their number senses' changes according to their grade, gender and mathematics performance. Results of this study showed that their number senses are seriously low (Kayhan Altay, 2010). The results also revealed that most commonly used method was rule-based methods on each component of number sense questions.

Aim of the Study

One component of number sense, "Understanding of the general meaning of numbers" underlies the importance of 10-number system which consists of whole numbers, fractions, and decimals. Decimal numbers, which are used in daily life as much as in science, have been recognized for some time to be a significant source of learning and teaching difficulties (Stacey et al., 2001). According to these studies, misconceptions of decimal numbers can be listed as below (Resnick et al., 1989; Widjaja, Stacey, & Steinle, 2011; Yılmaz, 2007)

- Mistakes in reading and writing decimal numbers,
- Mistakes in locating a decimal number on numerical axis,
- Misconceptions in the relation between the number of decimal digits and the actual value of that decimal number. (In general, it is thought erroneously that the decimal number increases as the number of decimal digits increases.)
- Fallacy of 'multiplying a number with a decimal number increases it' while 'dividing decreases',
- Mistakes in locating a negative decimal number on a numerical axis.

Sowder and Markovits (1989) stated that meaningful understanding of the size of fractions and decimal numbers can help students in developing number sense in general. When students' misconceptions are taken into consideration, it can be stated that students' number senses are not sufficient. For instance, a student whose number sense is not improved can normally think that 0.897 is greater than 0.9 just by looking at their number of decimal digits. On the other hand, a student whose number sense is high can easily say that the second number is greater than the first one. Therefore, as long as students develop their number senses, misconceptions that they have about decimal numbers will be solved as a result.

The aim of this study is to examine students' number sense about decimal numbers. In accordance with this purpose, changes of number senses about decimal numbers will be investigated according to its components, students' grades and genders, and achievement in mathematics. By this study, we believed that we will contribute to the literature about number senses in Turkey and also our students' current profiles will be shown in an international scale.

Prior to move to methodology, explanation of number sense and decimal numbers in Turkish curriculum will be helpful in understanding of students' profiles in Turkey.

Number Sense and Decimal Numbers in Curriculum

After the revision made in 2005 by the Turkish Ministry of National Education (MEB), some changes were made. After these changes take place, although the term of number sense is not included in the curriculum, its effect can be realized. Using abilities like estimation, mental computation, and flexible thinking effectively is one of the aims of this program (MEB, 2009). Although number sense's importance was emphasized with this aim, it wouldn't be wrong to state that necessary actions were not taken so as to make students to have number sense (Umay, Akkuş, & Paksu, 2006)

Decimal numbers are included in the curriculum starting from 4th grade. Acquisitions about this subject happen mostly on 6th grade. For example, it is expected from 6th grade students to have the ability of comparing decimal numbers in terms of their size, estimation of the solution of computations with decimal numbers and knowing how to round a decimal number (MEB, 2009).

Methodology

Study is designed with a survey method which aims to describe a situation in the past or current as it is (Karasar, 2000). Quantitative data of the study were obtained via Number Sense Test about Decimal Numbers (NSTDN) and qualitative data were obtained via interviews.

Sample

573 students from 6th, 7th and 8th grade were participated in the study (284 girls and 289 boys). Study was carried in different regions of Turkey at 19 classes in 6 different schools. These schools are settled on different regions with respect to economical status. Distribution of the students according to their grades and gender can be seen in Table 1.

Table 1. Distribution of students according to their grades and gender

	6th Grade	7th Grade	8th Grade	Total
Girls	49	120	115	284
Boys	62	129	98	289
Total	111	249	213	573

In the study, it is tried to get an idea of the question solution ways of the students, along with their number sense test performances. Therefore semi-structured interviews were given to students about how they solve the questions in the test. 3 students from each grade level, 9 in total, were selected. Maximum diversity and easy accessibility case modeling among the multi-purpose sampling models is used at the selection of these students. Students' nick names, genders, test scores, and their final marks from mathematics course are shared in Table 2.

Table 2. Students' nick names, genders, grades, NSTDN scores, and mathematics achievement

Nick Name	Gender	Grade	NSTDN Scores	Mathematics Achievement
Hüseyin	B	6	8	3
Melike	G	6	5	1
Aynur	G	6	7	4
Emine	G	7	7	1
Ulaş	B	7	10	3
Enes	B	7	10	5
Hakan	B	8	11	5
Beyza	G	8	11	4
Alim	B	8	7	3

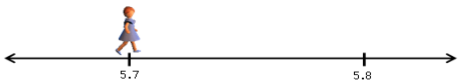
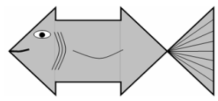
Instrument

In this study, NSTDN was used which was prepared by the researchers. While the test was being prepared, literature search was made to examine components of number sense. After the examination, it was decided to prepare questions that would be appropriate for components. Initial test consisted of 20 questions. After collaborating with some experts and considering their suggestions a pilot study was conducted. Two mathematics teachers and two mathematics educators examined the questions and decided whether the questions belonged to any component of number sense or not. When there is a split in the ideas, question was argued and common consent was obtained.

After the pilot study, 4 questions were excluded from the test. Final version of the test includes 16 questions, 4 different questions from 4 different components. 14 of them are multiple choice questions each of which had four different answer choices and only one of them was the correct answer. The remaining 2 questions were open-ended questions. To test the reliability of the test, Kuder-Richardson 20 coefficient was examined and its reliability was calculated to be 0.71.

Number sense components that are adapted to decimal numbers and a sample question for each component are listed in the Table 3.

Table 3. Sample questions from NSTDN

Number sense component	Sample question
Understanding the meaning of decimal numbers	The girl in the figure starts to walk from point 5.7 and after 10 steps, she reaches to point 5.8. Write down where she would be 4 steps after she started walking? 
Recognizing relative size of decimal numbers	There are three candles, having the same length and different colors each. Blue one melts with a ratio of 0.3 cm/min., red one with 0.30 cm/min., and yellow one with 0.300 cm/min. Without any detailed calculations, find out which one blows out first?
Being able to use benchmark appropriately	If the tail surface area of the below fish equals to 0.254 br^2., figure out which one can be the total surface area of this fish? 
Being able to judge the reasoning of a computational result that includes decimal numbers.	A bottle can take 0.6 lt. milk. Without any detailed calculations, find out how many bottles are required for 3.2 lt. milk?

NSTDN is used as an interview instrument at the same time. The questions in the test are shown respectively to the students and they are required to explain how they have solved these questions. Thus, it is tried to achieve the thinking process and strategies of the students during solving the questions. The findings obtained from the interviews have provided an idea regarding the solution strategies (number sense based or rule based) strategies of the students, who have participated to the interview. For each component of the number sense, one sample question is given and the solution strategy applied by the students at the respective question is examined in the study.

Procedure

Study was done in the spring semester of 2010-2011 using the improved version of the test that included 16 questions. It was given in a 40-minutes lecture hour. Before the test started, the teacher informed the students about the instructions of the test. The students were told that they shouldn't use any pen or pencil to make written calculations; they should just do mental computations and estimate the result. Also by stating "without detailed calculations", it was tried to prevent written calculations in the test. It was forbidden to pass a question or move into the next page until the previous page was completed. Thus, it is tried to provide that the students spend equal time for the questions and answer accordingly.

In addition to above, additional to students grades, gender etc., it was also asked that their mathematic achievement, because in the study, one of the main aims was to investigate the relation between mathematical achievement and the level of number sense about decimal numbers. To specify students' achievement, 2010-2011 Fall semesters' math grade marks were used. Marks were in the scale of 1 to 5.

After the implementation of the test, random students were selected. These students were interviewed one by one. Interviews took approximately 25 min. At the beginning of the interview, researcher took student's consent for approval of the interview and stated the aim of the interview. In order to make students' answers more clear, researcher used "can you explain this more" or "did you mean this" type of questions. Interviews were recorded with permission of students'.

Data Analysis

Data obtained via NSTDN were examined as qualitative data. Correct answers were labeled as 1 while incorrect ones were labeled as 0. On the analysis part, statistics was used. Frequency, average, standard deviation, t-test, one-way ANOVA and Pearson's correlation coefficient were used.

In the process of analyzing the semi-structured interview, first the interviews are converted into texts. The obtained transcripts are analyzed by content analysis and coded according to the question solution strategies of the students. The solution strategies are coded in two ways as number sense based and rule based. Methods like using measurement references (as 1, 0.5 etc.), estimation, mental arithmetic, composition/decomposition of numbers are used at number sense based strategies. If the student applies the rules of standard written algorithms, the strategy is coded as rule based.

Results

In this section, qualitative and quantitative data obtained from NSTDN and the interviews with students will be analyzed.

NSTDN Points According to Components of Number Sense and Grades

Firstly, students' scores of NSTDN were examined. With respect to number sense components and class levels, test results can be found in Table 4.

As it can also be seen in Table 4, students' results from number sense are really interesting. The average NSTDN score for 6th grade students is 5.19; 6.43 at 7th grade and 7.12 at 8th grade level. Students' grades have a positive factor. Although the best result was obtained at 8th grade students, they couldn't solve more than half the entire test. Students' scores show that secondary-stage students cannot use number sense about decimal numbers sufficiently.

When all four components of number senses are taken into account, in all grades, students got the worst scores on the component "*being able to judge the reasoning of a computational result that includes decimal numbers*". Their success percentage was 26.4% at this component. Components that students were successful can be listed as follows: "*understanding the meaning of decimal numbers*" and "*recognizing relative size of decimal numbers*". All students obtained the most success at the component "*being able to use benchmark appropriately*" 49.3% success was obtained in this component.

Table 4. NSTDN Points according to components of number sense and grades

Components of Number Sense	6th Grade			7th Grade			8th Grade			Total		
	X	SD	%	X	SD	%	X	SD	%	X	SD	%
Understanding the meaning of decimal numbers	1.26	.988	31.3	1.57	1.087	39.4	1.97	1.153	49.1	1.66	1.124	41.4
Recognizing relative size of decimal numbers	1.43	.959	35.5	1.79	1.153	44.7	1.87	1.081	46.8	1.75	1.101	43.7
Being able to use benchmark appropriately	1.72	1.011	42.7	1.98	1.016	49.5	2.10	.964	52.4	1.97	1.003	49.3
Being able to judge the reasoning of a computational result that includes decimal numbers.	.77	.805	19.3	1.09	.926	27.2	1.17	.968	29.3	1.06	.931	26.4
Total	5.19	2.384	32.2	6.43	2.867	40.2	7.12	2.790	44.4	6.44	2.831	40.2

The highest possible score was 16.

Change of Number Sense According To Grades

When all grades on secondary-stage are taken into consideration, success increases when grade level increases. In order to see whether this is significant or not, one-way ANOVA is conducted. Results are given in Table 5.

Table 5. Comparison of number sense according to grades

	Sum of Squares	Df	Mean Square	F	p*
Between Groups	271.438	2	135.719	17.932	.000
Within Groups	4313.968	570	7.568		
Total	4585.407	572			

* p is significant at the 0.05 level.

As it can be seen in Table 5, NSTDN and grade levels have a strong relationship [$F_{(2\text{-}570)}$=17.93, $p < 0.05$]. In order to understand on which classes his difference happens Scheffe's test was conducted.

Table 6. The result of Scheffe's test

Grades	Mean Difference	Std. Error	p*
6-7	-1.237	.314	.000
7-8	-.692	.257	.000
6-8	-1.928	.322	.027

* p is significant at the 0.05 level.

According to results, there are significant differences between 6th and 7th grade students, 7th and 8th grade students and 6th and 8th grade students. This shows that when grade level increases, number sense of decimal numbers increases.

Change of Number Sense According To Gender

T-test was conducted to demonstrate that students' number senses of decimal numbers can vary with respect to gender. Analysis results are shown in Table 7.

Table 7. Comparison of number sense according to gender

Gender	N	X	S	sd	t	p*
Girls	284	6.21	2.719	571	-1.919	.055
Boys	289	6.67	2.925			

* p is significant at the 0.05 level.

As it can be seen in Table 6, male students got 6.67 while female students got 6.21 average score. Although males have higher scores, according to statistics, this difference is not significant [$t_{(571)} = -1.91$, $p < 0.05$]. When all students are considered, there is no significant difference in terms of gender.

Relationship between Number Sense and Mathematical Achievement

In order to compare students' number senses about decimal numbers and their mathematical achievement, their NSTDN scores and math grade marks were used and Pearson' correlation coefficient was used. According to analysis' results, an average and positive correlation (0.430) was obtained. When their success on mathematics increases, number sense of decimal numbers also increases.

Example Questions about Number Sense On NSTDN and Students' Answers to These

In this section, sample questions from each component of NSTDN are shared and correct answer percentages of all students for each question are stated. In addition, the students' answers are also shared to understand the reasons. These questions were selected in accordance to their difficulties.

The following question (Table 8) a conceptual type of question for which the students were expected to know the fact that there always exits a number between two different decimal numbers. The answers were unexpected. Highest success obtained at 8th grade with 20%. Most of the students selected the choice A or B wrongfully as their answers. B was the most selected choice in 6th and 7th grades while A was the most selected one in 8th grade classes.

Table 8. An example from first component of number sense

Component of Number Sense	Question	Correct Percentage
Understanding the meaning of decimal numbers	How many different decimal numbers are there between 1.52 and 1.53? **A)** 0 **B)** 10 **C)** 100 **D)** Infinitely many	6th grade %18.0 7th grade %18.1 8th grade %20.1

Only 3 out of 9 students (Ulaş, Beyza, Alim) gave the correct answer for this question. Most of the students who were wrong couldn't give a satisfactory answer. This shows that students have no sufficient knowledge about the subject. 3 students said "since there is no integer between 52 and 53, there shouldn't be any number between two decimal numbers."

"Because 0.52 is a decimal numbers. There is no number up to 0.53." (Hakan)

"Since they are consecutive numbers, no number exists." (Enes)

Students who were correct about there exist infinitely many numbers between two decimal numbers gave the following reasoning:

"We can make as much period as we can between these." (Ulaş)

"We can add as much number as we can to these numbers. Then it is infinite." (Beyza)

Low ratio of success in this question and students' answers demonstrated that they have serious misunderstanding about the concept of decimal numbers.

In this following question, it was expected from students to compare two decimal numbers. When students' grade level increases, the percentage of correct answers also increases in this question. All of 8th grade classes have 51% success on this question.

Table 9. An example from second component of number sense

Component of Number Sense	Question	Correct Percentage
Recognizing relative size of decimal numbers	Given that k is 0.89 and m is 0.9, which of the following ordering is true? **A)** $k > m$ **B)** $k = m$ **C)** $k < m$ **D)** Impossible to tell without working it out.	6th grade %33.3 7th grade %40.8 8th grade %51.1

Students who have the idea of "longer is larger" think that if the number of decimal digits is higher in a decimal number, that means that number is larger than the other one (Steinle & Stacey, 1998). Other than the correct answer, A was the most selected choice with 33.8% and that can demonstrate the situation. Examples of this situation were also experienced through interviews with students. Student named Alim has the following explanation:

"(In this question) 90 would be larger but it was 9. Since it is 89, it is larger."

Another question comes from another type-of-component, it was asked to order 4 numbers with respect to their sizes from the smallest to the largest (3.46, 5.441, 3.87, and 4.949). Other than 2 of those, numbers have different integer parts. Number of decimals of these numbers is the same (3.46, 3.87). Students were successful with a percentage of 83.6% in this question. The huge difference between these two questions is another demonstration of the students' insufficiencies.

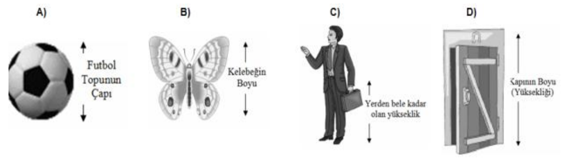
Due to the fact that 0.9 equals to 0.90, 3 of the interviewed students stated that "k is less than m". Students' who have used rule-based approach explanations are as follows:

"I looked at the number after the comma. One look to 9 and one look to 8. So, m is larger. Our teacher told this rules last year." (Hüseyin)

"If I move the comma to right by one decimal, one will become 9 and the other 8.9. Since 9 is larger, m is larger too." (Beyza)

When NSTDN results were examined, the component of "being able to use benchmark appropriately" was the one that students were most successful (Table 4). Questions given in Table 10 are included in this component and it was the question that the students were least successful. Due to low success rate, this question was not examined. Rather than giving students a benchmark in these questions, they are expected to define a new benchmark from their daily lives and use that one. Each class level's success ratios are pretty low at this question. For the other questions, the success rate increases as the class level increases too.6th grade students' success was 11.7%, 7th grade students' 21.7% and 8th grade students' were 24.2% for this question. Only 4 out of 9 students who were interviewed could give the correct answer.

Table 10. An example from third component of number sense

Component of Number Sense	Question	Correct Percentage
Being able to use benchmark appropriately	Which one of the following objects has nearly 0.2 meters length? 	6th grade %11.7 7th grade %21.7 8th grade %24.2

The most selected choice in this question was B which includes a butterfly figure. Two root reasons are obtained after the interviews with students. First one is that 0.2 is a very small number for students. Students who have defended this idea stated the following:

> *"Door's length is almost equals to man's body length. Ball is larger than a butterfly. Smallest one is butterfly." (Melike)*

> *"Shortest one is butterfly. Door would be really long." (Emine)*

Second reason of why students picked choice B is the mistakes that they have made during converting a unit into another. Hüseyin's answer can be considered as an example to this situation:

> *"This number is 2 cm. That is this. (Pointing at...) That's the butterfly."*

2 answers from students who were correct on this question are the following:

> *"A butterfly cannot be 20 cm. Door or a man impossible. Than answer is ball." (Enes)*

> *"By adding zeros to its end, I converted it into cm. Answer is A. Because butterfly cannot exceed 20 cm." (Ulaş)*

Table 11. An example from fourth component of number sense

Component of Number Sense	Question	Correct Percentage
Being able to judge the reasoning of a computational result that includes decimal numbers.	Ayşe calculated the result of 0.495 x 248 by using a calculator. Then, she realized that she did not put the point in the decimal number while using the calculator. Without making a multiplication process, guess where the point should be put. **A)** 1.2276 **B)** 1227.6 **C)** 12.276 **D)** 122.76	6th grade %10.8 7th grade %22.8 8th grade %26.0

In this question, it is expected from the students to multiply a decimal number with an integer so that they can decide where to put the comma. In all class levels, this question has a low success. 6th grades success was 10.8%. 2 out of 9 interviewed students were correct on this question.

Most selected choice was C with 33.7%. This demonstrated that students were using rule-based approaches rather than number sense while solving the question. Enes who used rule-based approach explained himself by the following:

> *"Decimal numbers are multiplied as if they are integers. As much as number of decimal part, comma is put to that much left from last digit of obtained result. There were 3 decimal digits, so I put third place from right."*

Melike and Hüseyin used the similar techniques but they were mistaken. Here are their explanations:

> *"O stays at the beginning. When we multiply all those numbers and they go down, comma goes down too. Since there exists only one "0", answer is A." (Hüseyin)*

> *"There is a comma after "0". Since there is only one before comma." (Melike)*

Students who have used number sense while solving questions considered 0.495 as 0.5 and they found the half of the number when they multiply with that number. As it can be seen on success ratios, only a few students gave D choices as their answers. Explanations of the students who were right are the following:

> *"I considered first question as 0.5. 248 times 0.5 makes approximately 122." (Hakan)*

> *"I assumed 0.495 as half of "1". When we multiply it with 200, it exceeds 100. Result would be kind of 122.76." (Ulaş)*

Discussion and Conclusion

Though there are conducted diverse studies on number sense at students in the foreign literature, the number of these types of studies in our country is limited. The purpose of this conducted study is to examine number sense on the subject of decimal numbers of the secondary stage students in Turkey and to reveal the existing situation. The findings of the study show that 32.2% of the 6th grade students, 40.0% of the 7th grade students and 44.4% of 8th grade students have used their number sense on decimal numbers. This result is parallel to the finding that the number senses of students in the secondary stage are rather low, revealed in studies conducted in our country and abroad (Harç, 2010; Kayhan Altay, 2010; Yang, Li & Lin, 2008; Yang, 2005a). There may be several reasons for such a performance of students at the usage of the number sense. These possible reasons are indicated below.

1. The strategies resulted from the conducted interviews with the students have revealed that the students remained stick to rule based strategies and generalizations. The students are used the rules they memorized instead of perceiving the numbers and the real meanings of the processes and applying this flexibly. It is noted also in the literature that students tend to solve problems rule based (Reys et al., 1999; Reys & Yang, 1998; Tsao, 2004).

The reasons for that students in Turkey tend to use rule based strategies is lead by annually held national examinations and the schools and training centers, which prepare the students for these examinations. That what is expected from the students at these examinations is that they achieve a definite result in a short time. This situation urges students to use short solution ways they memorized, rather than using alternative thinking ways. And the teachers teach the students rules, which will provide the students to achieve the result in short time rather than providing a conceptual education in order to make the students succeed at these examinations. If it is desired that students use number sense strategies more, so open end and estimation questions need be included to the performed examinations.

Another reason for that students tend to use rule based strategies is that number sense is not sufficiently regarded in the curriculum and that thus the students are not taught how to use number sense strategies. It is revealed in performed studies that number sense is teachable and able to be developed by different teaching methods (Irwin & Britt, 2005; Markovits & Sowder, 1994; Yang, 2003, Yang, Hsu, & Huang, 2004; Yang & Wu, 2010). And this indicates the necessity to include number sense into the mathematics curriculum. The students will develop their mental arithmetic and estimation skills and discover their own solution strategies by the learned number sense strategies. Therefore, the mathematics curriculum needs to be reorganized such to include number sense.

2. The insufficiency of the conceptual knowledge of students in terms of decimal numbers can be shown as a second reason. As also mentioned in performed studies (Arslan & Ubuz, 2009; Bilgin & Akbayır, 2002; Resnick et al., 1989), the students are hard put to understand the decimal numbers subject and have misconceptions about the subject. For instance, success ratio of the question given in Table 8 also shows that students could not totally get the idea of decimal numbers and they have some misconceptions about decimal numbers. And, for instance, the correct answer ratio for the question in Table 8 (How many different decimal numbers between 1.52 and 1.53?) reveals that the students were not able to configure exactly the meaning of decimal numbers and that there are deficits in their knowledge.

The resulting situation indicates the necessity for a very good perception of the meanings of numbers and processes. For instance, during teaching multiplication to a student, not the classical rules for multiplication, but conceptual knowledge that multiplication is actually a repeated addition needs to be taught first. A student, knowing the conceptual meaning of multiplication, will be able to use the calculation as he/she wants to and to use his/her own multiplication strategies. A student with a high number sense would, instead of using the written multiplication algorithm with paper and pen when calculating 56 x 4, would rather deem the calculation as (50 x 4) + (6 x 4) and be able make this rapidly in his/her mind.

3. Another reason of the obtained results is that teachers may have deficiencies about number sense. Yang, Reys and Reys (2009) found that teachers in Taiwan are not capable of using number sense and they tend to use written methods. In our country, since there is no study about the topic, it is impossible to reach a definite conclusion like this. However, since teachers are educated in a similar environment, it can be deduced that they also tend to use rule-based strategies. For a teacher who uses rule-based strategies and thinks that number sense is not required it is impossible to teach number sense to his/her students. Teachers who will implement the program should be motivated and should be taught how to prepare an environment for the students so that they can be successful during education of number sense.

The least successful component was "*being able to judge the reasoning of a computational result that includes decimal numbers*" in 6th, 7th and 8th grade students' study. This result is also parallel to Yang & Li (2008) and Yang, Li & Lin (2008)'s results. The researchers from Taiwan emphasize this failure. On the other hand, *"Being able to use benchmark appropriately"* component is the one that students obtained the best success. This result is consistent with Harç' (2010) study. According to Harç, the main reason why students obtained the best result from this component is that they are kind of visual questions. Estimating the objects' weights and sizes are enough to solve these questions. Not using any algorithm or rule may be considered as a reason of this success.

According to research results, it was observed that age has a positive effect on number sense of decimal numbers. This result also shows consistence with other results (Aunio, Ee, Lim, Hautamäki, & Van Luit, 2004; Aunio et al., 2006; Yang, 1995) and inconsistence with other studies (Kayhan Altay, 2010). As algorithmic written and rule based methods are learned, students internalize them more (Markovits & Sowder, 1994). This situation reduces the use of number sense based strategies of students.

Research shows that gender does not have a significant effect on number sense. This result also shows consistence with other results in the literature (Aunio, et al., 2004; Aunio, et al., 2006; Harç, 2010; Kayhan Altay, 2010; Yang, 1995; Yang & Li, 2008). When NSTDN grades of students and their mathematical achievements are compared, moderate level correlation was found. Similar studies can be found in the literature (Kayhan Altay, 2010; Yang, et al., 2008). This situation is a demonstration of significant relation between mathematical success and number sense. This moderate correlation indicates that even the student in this study having high mathematics achievement is not able to sufficiently use number sense base strategies at problem solving. It can be said that number sense education is in adequate.

This study frequently mentioned that studies in Turkey are insufficient. Number of studies which will investigate the number sense about decimal numbers of students who are in the secondary stage of primary education should be increased. It will be beneficial to investigate primary stage students or high-school students' profiles about the topic. Also teachers' deficiencies should be examined about number senses. It is hoped that this study will be an initiator for future studies and will be guidance for them.

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