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PORTFOLIO SELECTION WITH HIGHER MOMENTS: A POLYNOMIAL GOAL PROGRAMMING APPROACH TO ISE–30 INDEX

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Abstract

The aim of this paper is to propose a portfolio selection model which takes into account the investors preferences for higher return moments such as skewness and kurtosis. In the presence of skewness and kurtosis, the portfolio selection problem can be characterized with multiple conflicting and competing objective functions such as maximizing expected return and skewness, and minimizing risk and kurtosis, simultaneously. By constructing polynomial goal programming, in which investor preferences for skewness and kurtosis incorporated, a Turkish Stock Market example will be presented for the period from January 2005 to December 2010.

Keywords: Mean-Variance-Skewness-Kurtosis Portfolio Model, Polynomial Goal Programming, Risk Preference.
Jel Classification: C44, G11

Özet

Bu makalenin amacı, çarpıklık ve basıklık gibi yüksek getiri momentleri için yatırımcının tercihlerini göz önüne alan bir portföy seçimi modeli önermektir. Çarpıklık ve basıklığın varlığında, portföy seçimi problemi, eş zamanlı olarak beklenen getiri ve çarpıklığın maksimizasyonu ile risk ve basıklığın minimize edilmesi gibi birbiri ile çelişen ve rekabet eden amaç fonksiyonları ile karakterize edilir. Polinomsal hedef programlama oluşturarak, Ocak 2005-Aralık 2010 periyodu için Türk Borsası'nda bir örnek sunulacaktır.

Anahtar Kelimeler: Ortalama-Varyans-Çarpıklık-Basıklık Portföy Modeli, Polinomsal Hedef Programlama, Risk Tercihi.
Jel Sınıflaması: C44, G11

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1. INTRODUCTION

In the modern portfolio theory, the mean-variance model which is minimizing risk for a given level of expected return, or equivalently, maximizing expected return for a given level of risk originally introduced by Markowitz (1952) and has gained widespread acceptance as a practical tool for portfolio optimization. Since the seminal work of Markowitz, most contributions to portfolio selection are based only first two moments of return distribution.

In Markowitz's framework, it is assumed that asset returns follow multivariate normal distribution. This means that the distribution of asset return can be completely described by the expected value and variance. However empirical finance has shown that the distribution of individual asset returns sampled at a daily, weekly or monthly frequency exhibit negative skewness and excess kurtosis so is not well described by a normal distribution. In the presence of negative skewness, negative return has higher probability than positive return. In addition, if a distribution of portfolio return is positively skewed, it indicates that poor returns occur frequently but losses are small, whereas very high returns occur less frequently but are more extreme. Furthermore, the kurtosis can reflect the probability of extreme events. Excess positive kurtosis, or leptokurtosis indicates that a distribution of return has fatter tails than a normal distribution, i.e., it indicates a higher probability of very high and very low returns would be expected than the normal case. This departure from normality means that higher moments of the return distribution are necessary to describe portfolio behavior. When the skewness and kurtosis are significant, if we look at only the mean and variance under the normality assumption for the return distribution, we may underestimate the risk and this leads to obtain an inefficient portfolio. Thus the mean-variance model proposed by Markowitz is inadequate for optimal portfolio selection problem and higher moments can not be neglected.

One of the problems of extending the mean-variance framework to higher moments for portfolio selection is that it is not easy to find a trade-off between the four objectives because in the presence of skewness and kurtosis, the problem turns into a nonconvex and nonsmooth multiobjective optimization problem. Thus, many researches on portfolio selection largely concentrate on the first three moments and kurtosis is neglected by most of researchers. In addition, most of models only consider the distribution of asset return but other factors, such as investor's risk preferences and trading strategies, are not taken into account.

To tackle these problems, many approaches have been proposed. One of the efficient ways to solve this task is polynomial goal programming method. An important feature of polynomial goal programming problem is the existence of optimal solution since feasible solution always exists. The other important features of this method are its flexibility of incorporating investor preferences and its simplicity of computational requirements. As a result, this study extends the work of Lai *et al.* (2006) by utilizing polynomial goal programming, which incorporates investor preferences for skewness and kurtosis.

In summary, the main focus of this study is to propose a mean-variance-skewness-kurtosis model for portfolio selection problem based on investor's risk preferences by constructing polynomial goal program. The paper is organized as follows: Section 2 provides a brief review of literature. In section 3, the theoretical framework of the portfolio selection problem with higher moments is discussed. The numerical results are illustrated in section 4. The final section concludes the study while some computational details are relegated to an appendix.

2. LITERATURE REVIEW

Since Markowitz (1952, 1959) proposed the mean-variance portfolio model, numerous studies of portfolio selection have focused on the first two moments of return distributions for portfolio decisions. In his framework, return distribution is assumed to be normal or utility function only depends on first two moments, i.e., utility function is quadratic. It is well known that financial series are non-normal. Also many empirical evidences suggest that asset returns tend to be asymmetric and leptokurtic, that is, more peaked and fatter tailed than the normal distribution: See Mandelbrot (1963), Fama (1965), Blattberg ve Gonedes (1974), Kon (1984), Mills (1995), Campbell (1997), Peiro (1999), Harvey and Siddique (1999, 2000), Premaratne and Bera (2000). However, many researchers argued that the higher moments can not be neglected unless there is a reason to believe that the asset returns are normally distributed or the utility function is quadratic, or that the higher moments are irrevelant to the investor's decision: See Samuelson (1970), Arditti (1971), Rubinstein (1973), Scott and Horvath (1980), Lai (1991), Konno and Suzuki (1995), Chunhachinda *et al*. (1997), Prakash *et al*. (2003), Lai *et al*. (2006).

Moreover, Hanoch and Levy (1970) pointed out that that the quadratic utility function implies increasing absolute risk aversion which is contrary to the normal assumption of decreasing absolute risk aversion. Levy and Sarnat (1972) also shows that the assumption of a quadratic utility function is appropriate only for relatively low returns (Chunhachinda *et al.*,1997).

Furthermore, when the investment decision is restricted to a finite-time interval, Samuelson (1970) showed that the mean-variance efficiency becomes inadequate and higher moments become relevant to the portfolio selection (Lai, 1991).

In general, investors will prefer high values for odd moments and low ones for even moments. The former can be seen as a way to decrease extreme values on the side of losses and increase them on the gains' (Athayde and Flores, 2004). Scott and Horvath (1980), investigated the use of higher moments in portfolio analysis by determining direction of preference of moments. They showed that preference is positive (negative) for positive values of every odd central moment and negative for every even central moment for investor who is consistent in direction of preference of moments.

As a result, in some recent studies the concept of mean-variance framework has been extented to include the skewness and kurtosis of return in portfolio selection (Yu *et al.*, 2008).

The importance of skewness in return distribution is introduced by Arditti (1967, 1971) in the pricing stocks. Kraus and Litzenberger (1976), came up with three parameter capital asset pricing model (Premaratne and Bera 2000). Lai (1991), Chunhachinda (1997) and Prakash *et al*. (2003) showed that the incorporation of skewness into the investor's portfolio decision causes a major change in the constructing of the optimal portfolio.

In fact, kurtosis which reflects the probability of extreme events is neglected for a long time by most researchers. As the dimensionality of the portfolio selection problem increases, then it becomes difficult to develop geometric interpretation of the quartic portfolio efficient frontier and to select the most preferred portfolio among boundary points (Jurczenko *et al.*, 2006). Mandelbrot (1963), was probably the first to take into account excess kurtosis in financial data as he noted that the price changes were too peaked and thick-tailed than normal distribution (Premaratne and Bera 2000). In spite of the considerable empirical literature now taking into account this fact, financial theory has been reluctant in incorporating higher moments such that kurtosis in its developments (Athayde and Flores, 2004). Jean (1971), extends the portfolio analysis to three or more parameters and derives the risk premium for higher moments (Chunhachinda *et al.*,1997). Fang and Lai (1997), first introduced kurtosis to develop capital asset pricing model as well as skewness. Jondeau *et al*. (2006), introduced the kurtosis into the portfolio selection problem through utility function (Qi-fa *et al*., 2007). Also, there are some researches look for the analytical solution in the mean-variance-skewness-kurtosis space: See Athayde and Flores (2004), Adcock (2005), Jurczenko and Maillet (2005b). Furthermore, Jondeau and Rockinger (2003, 2005), Jurczenko and Maillet (2005a) used Taylor series expansion of the investors' objective functions to determine optimal portfolio (Jurczenko *et al*., 2006).

In the presence of skewness and kurtosis, the portfolio selection problem turns into a nonconvex and nonsmooth optimization problem which can be characterized with multiple conflicting and competing objective functions such as maximizing expected return and skewness, and minimizing risk and kurtosis, simultaneously. To solve this complicated task, different approaches have been proposed in the literature and one of the efficient way is applying polynomial goal programming (PGP) which investment strategies and the investor's preferences should be included.

PGP was first introduced by Tayi and Leonard (1988). After, Lai (1991) applied PGP to portfolio selection and explored incorporation of investor's preferences in the construction of a portfolio with skewness. Similarly, Leung *et al*. (2001) provided PGP to solve mean-variance-skewness model with the aid of the general Minkovski distance. In the mean-variance-skewness framework, also Chunhachinda *et al.* (1997), Wang and Xia (2002), Sun and Yan (2003), Prakash *et al.* (2003) used PGP to construct optimal portfolio. Lai *et al.* (2006) augmented the dimension of portfolio selection in PGP from mean-variance-skewness to mean-variance-skewness-kurtosis. More recently, incorporating higher moments such as skewness and kurtosis, PGP has subsequently been used as an efficient way by Qi-fa *et al.* (2007), Taylan and Tatlıdil (2010), Mhiri and Prigent (2010) for efficient portfolio and also Davies *et al.* (2009) and Berenyi (2005) for effiecient funds of hedge funds.

3. PORTFOLIO SELECTION WITH HIGHER MOMENTS

In this section, we consider the problem of an investor who selects optimal portfolio from *n* risky assets in the presence of skewness and kurtosis of return distribution which is a trade-off between competing and conflicting objectives, i.e., maximizing expected return and skewness, while minimizing variance and kurtosis, simultaneously. As Lai (1991) notes that there are some standard assumptions in portfolio selection, we accept these assumptions except some minor points such that:

i) investors are risk-averse individuals who maximize the expected utility of their end-of-period wealth,
ii) there are *n* risky asset and investor does not have access to a riskless asset implying that the portfolio weights must sum to one,
iii) all asset are marketable, perfectly divisible,
iv) the capital market is perfect, there are no taxes and transaction costs,
v) short selling is not allowed, implying that portfolio weights must be positive.

Our major interest is to determine the investment strategy of the investor among different preferences and the investment weight of each asset which should be included within the mean-variance-skewness-kurtosis framework.

Let's denote portfolio return by R_p, $\mathbf{R} = (R_1, R_2, ..., R_n)$ is the return vector, R_i is the rate of return of ith asset. Wealthes are allocated to *n* assets by the weights $\mathbf{X} = (x_1, x_2 ..., x_n)$, x_i is the proportion invested in the *i*th asset when the best trade-off is found. The mean, variance, skewness and kurtosis of the rate of return R_i on asset *i* are assumed to exist for all risky assets *i*, *i*=1,2,…,*n* and denoted by $\bar{R}_i$, σ_i^2, s_i^3, k_i^4; respectively. Then, the first four moments of portfolio return R_p can be computed as:

$$Mean = \mathrm{E}(R_p) = \mathbf{X}'\bar{\mathbf{R}} = \sum_{i=1}^{n} x_i \bar{R}_i \tag{1}$$

$$Variance = \sigma^2(R_p) = \mathbf{X}'\Sigma\,\mathbf{X} = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}\ , (i \neq j) \tag{2}$$

$$Skewness = S^3(R_p) = \mathrm{E}(\mathbf{X}'(\mathbf{R} - \bar{\mathbf{R}}))^3 =$$

$$\sum_{i=1}^{n} x_i^3 s_i^3 + 3\sum_{i=1}^{n}\left(\sum_{j=1}^{n} x_i^2 x_j s_{iij} + \sum_{j=1}^{n} x_i x_j^2 s_{ijj}\right), (i \neq j) \tag{3}$$

$$Kurtosis = K^4(R_p) = \mathrm{E}(\mathbf{X}'(\mathbf{R}-\bar{\mathbf{R}}))^4 =$$

$$\sum_{i=1}^{n} x_i^4 k_i^4 + 4\sum_{i=1}^{n}\left(\sum_{j=1}^{n} x_i^3 x_j k_{iiij} + \sum_{j=1}^{n} x_i x_j^3 k_{ijjj}\right) + 6\sum_{i=1}^{n}\sum_{j=1}^{n} x_i^2 x_j^2 k_{iijj}\ ,\ (i \neq j) \tag{4}$$

where σ_{ij} is variance-covariance matrix; s_{iij}, s_{ijj} are skewness-coskewness (which measure curvelinear relationship); k_{iiij}, k_{ijjj}, k_{iijj} are kurtosis-cokurtosis matrices of the joint distribution of risky asset returns R_i and R_j and they are defined as follows:

$$\sigma_{ij} = \frac{1}{t}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(R_i - \bar{R}_i\right)\left(R_j - \bar{R}_j\right) \tag{5}$$

$$s_{iij} = \frac{1}{t}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(R_i - \bar{R}_i\right)^2\left(R_j - \bar{R}_j\right),\ s_{ijj} = \frac{1}{t}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(R_i - \bar{R}_i\right)\left(R_j - \bar{R}_j\right)^2 \tag{6}$$

$$k_{iiij} = \frac{1}{t}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(R_i - \bar{R}_i\right)^3\left(R_j - \bar{R}_j\right), k_{ijjj} = \frac{1}{t}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(R_i - \bar{R}_i\right)\left(R_j - \bar{R}_j\right)^3$$

$$k_{iijj} = \frac{1}{t}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(R_i - \bar{R}_i\right)^2\left(R_j - \bar{R}_j\right)^2, \tag{7}$$

(where t is the number of periods).

Then, the portfolio selection problem with higher moments can be formulated with following competing and conflicting objective functions:

$$(\mathrm{P1})\begin{cases}\text{Max} \quad \mathrm{E}(R_p) = \mathbf{X}'\bar{\mathbf{R}} \\ \text{Min} \quad \sigma^2(R_p) = \mathbf{X}'\Sigma\mathbf{X} \\ \text{Max} \quad S^3(R_p) = \mathrm{E}(\mathbf{X}'(\mathbf{R}-\bar{\mathbf{R}}))^3 \\ \text{Min} \quad K^4(R_p) = \mathrm{E}(\mathbf{X}'(\mathbf{R}-\bar{\mathbf{R}}))^4 \\ \text{s.t.} \qquad \mathbf{X}'\mathbf{I} = 1 \\ \qquad\qquad x_i \geq 0,\ i = 1,2,\ldots,n.\end{cases} \tag{8}$$

A general way to solve the multiobjective problem is to consolidate the various objectives into a single objective function. Because of the contradiction and possible incommensurability of the objective functions such as risk and return, it's often not possible to find a single solution where every objective function attains its optimum simultaneously. Generally, instead of single solution, a set of nondominated solutions is considered. In this case, subjective judgements of investor come into prominence.

As a result, the multiobjective problem involves two step procedures: First, a set of nondominated solutions which is independent from investor's preferences is developed. After, investor selects the most preferable solution among the given set of solutions. The second step can be accomplished by incorporating investor's preferences for objective functions into the construction of a polynomial goal programming. Consequently, portfolio selection with higher moments is a solution of PGP.

3.1 Solving Polynomial Goal Programming

A solution depending on investor preferences for objectives can be determined by constructing of a polynomial goal programming into which the specified investor's personal objectives are incorporated. Thus, we use this approach to combine our objectives into single one, and to solve (P1).

The main interest of polynomial goal programming can be defined as the minimization of deviations from ideal scenario set by aspired levels. The aspired level indicates the best scenario for a particular objective without considering other objectives. Hence , the aspired levels, M^*, V^*, S^*, K^*, can be determined by solving four independent subproblems, using linear and nonlinear programming:

$$(\text{SP1})\begin{cases}\text{Max} \quad \text{E}(R_p) = \mathbf{X}'\bar{\mathbf{R}} \\ \text{s.t.} \quad \mathbf{X}'\mathbf{I} = 1 \\ x_i \geq 0,\ i = 1,2,\ldots,n.\end{cases} \qquad (\text{SP2})\begin{cases}\text{Min} \quad \sigma^2(R_p) = \mathbf{X}'\Sigma\mathbf{X} \\ \text{s.t.} \quad \mathbf{X}'\mathbf{I} = 1 \\ x_i \geq 0,\ i = 1,2,\ldots,n.\end{cases}$$

$$(\text{SP3})\begin{cases}\text{Max } S^3(R_p) = \text{E}(\mathbf{X}'(\mathbf{R}-\bar{\mathbf{R}}))^3 \\ \text{s.t.} \quad \mathbf{X}'\mathbf{I} = 1 \\ x_i \geq 0,\ i = 1,2,\ldots,n.\end{cases} \qquad (\text{SP4})\begin{cases}\text{Min } K^4(R_p) = \text{E}(\mathbf{X}'(\mathbf{R}-\bar{\mathbf{R}}))^4 \\ \text{s.t.} \quad \mathbf{X}'\mathbf{I} = 1 \\ x_i \geq 0,\ i = 1,2,\ldots,n.\end{cases}$$

Let d_1, d_2, d_3, d_4 be the nonnegative goal variables which account for the deviations of expected return, variance, skewness and kurtosis from the aspired levels, M^*, V^*, S^*, K^*, respectively. In other words, the goal variables denote the amount of underachievement with respect to the best scenario. To minimize objective function, general Minkovski distance is often used in finance and economics. The computational form of Minkovski distance is:

$$Z = \left\{ \sum_{i=1}^{n} \left| \frac{d_i}{Z_i} \right|^p \right\}^{1/p} \qquad (9)$$

where Z_i is the basis for normalizing the ith variable. To incorporate investor's different preferences towards to the mean, variance, skewness, kurtosis of portfolio return into model, we introduce four parameters λ_1, λ_2, λ_3, λ_4, respectively. Since λ_i parameters represent the investor's subjective degree of preferences, the greater λ_i, the more important corresponding moment of portfolio return is to the investor.

In PGP, the objective function contains deviational variables between goals and what can be achieved and does not contain choice variables. Given the investor preferences, the multiobjective portfolio selection problem (P1) turns into single-objective problem by constructing the PGP model (P2) whose objective is to minimize deviations from ideal scenario set by aspired levels as follows:

$$(\text{P2}) \begin{cases} \text{Min} \quad Z = \left|\dfrac{d_1}{M^*}\right|^{\lambda_1} + \left|\dfrac{d_2}{V^*}\right|^{\lambda_2} + \left|\dfrac{d_3}{S^*}\right|^{\lambda_3} + \left|\dfrac{d_4}{K^*}\right|^{\lambda_4} \\ \\ \text{s.t.} \quad \mathbf{X}'\bar{\mathbf{R}} + d_1 = M^* \\ \qquad \mathbf{X}'\sum\mathbf{X} + d_2 = V^* \\ \qquad \mathrm{E}(\mathbf{X}'(\mathbf{R}-\bar{\mathbf{R}}))^3 + d_3 = S^* \\ \qquad \mathrm{E}(\mathbf{X}'(\mathbf{R}-\bar{\mathbf{R}}))^4 + d_4 = K^* \\ \qquad \mathbf{X}'\mathbf{I} = 1 \\ \qquad x_i \geq 0,\ d_i \geq 0,\ i = 1,2,\ldots,n \end{cases} \qquad (10)$$

The set of efficient portfolio consists of solutions of problem (P2) for various combinations of λ_i. In this study, we also obtained efficient portfolio for the mean-variance, and mean-variance-skewness case and compared to those of the mean-variance-skewness-kurtosis efficient portfolio.

4. EMPIRICAL RESULTS

In the light of earlier description, our analysis is based on two purposes:

i) to demonstrate the formulation of the polynomial goal programming for portfolio selection problem in four-moment space,
ii) to illustrate how portfolio selection will vary for investors with different investment strategies.

The sample data consists of monthly rates of return for 26 stocks from Istanbul Stock Exchange-30 Index in Turkish Stock Market for the period January 2005 through December 2010. The historical data are used to estimate the expected return, covariance and central comoments.

The empirical experiment employed in this study can be summarized in two main stages: First, the distributional properties are computed and normality test results are represented in Table 1. In addition, in Table 2, the stocks are ranked based on the coefficient of variation to provide some preliminary information. Secondly, the aspired levels are found by solving (SP1)-(SP4), as shown in Table 3. Then, by solving (P2) with PGP approach, optimal objective values and the trade-off between them are shown in Table 4. Moreover, the optimal weights of the stocks which should be included in portfolio are presented for the given investor's different preferences including also MV and MVS case in Table 5. All of the results are calculated on GAMS program.

For preliminary analysis, Table 1 lists the descriptive statistics of the rate of return of 26 stocks. Interestingly, while DENİZ has the highest value of mean and skewness, i.e., return, it also has the highest value of variance and kurtosis, that is, risk. The results of the normality of return distributions using the Jarque-Bera test are also provided in the last column. Since test results supports non-normality of return distribution, there is an evidence to construct portfolio including skewness and kurtosis.

Table 1. Descriptive statistics and normality test for rate of return distribution

Stock	*Variable*	*Mean*	*Variance*	*Skewness*	*Kurtosis*	*J-B Statistic**	*p-value*
AKBNK	x_1	1,882	189,614	0,806	4,868	15,23	0,00
ARCLK	x_2	1,563	226,188	0,398	4,287	5,727	0,06
DENİZ	x_3	**3,733**	609,699	**3,498**	18,687	737,597	0,00
DOAS	x_4	2,661	271,465	-0,349	2,79	1,329	0,51
DOHOL	x_5	0,739	170,357	0,554	3,656	4,143	0,13
DYHOL	x_6	0,452	363,36	0,53	4,196	6,385	0,04
EREGL	x_7	2,379	162,764	-0,232	2,73	0,718	0,70
FINBN	x_8	1,745	102,412	1,158	7,247	58,495	0,00
GARAN	x_9	2,682	208,699	0,395	3,012	1,561	0,46
HURGZ	x_{10}	0,014	276,257	0,424	3,499	2,418	0,30
ISCTR	x_{11}	0,98	141,844	0,4	2,906	1,618	0,45
ISGYO	x_{12}	0,657	148,502	-0,087	3,547	0,823	0,66
KCHOL	x_{13}	2,369	167,71	0,113	3,323	0,388	0,82
PETKM	x_{14}	1,353	137,457	-0,077	3,379	0,418	0,81
PTOFS	x_{15}	1,862	181,953	-0,168	4,138	3,516	0,17
SAHOL	x_{16}	1,758	202,707	0,582	3,818	5,064	0,08
SISE	x_{17}	1,404	140,435	-0,163	2,941	0,273	0,87
SKBNK	x_{18}	2,697	322,028	0,405	4,361	6,266	0,04
TCELL	x_{19}	1,558	**99,753**	-0,211	3,411	0,867	0,65
THYAO	x_{20}	3,114	170,82	0,04	2,705	0,233	0,89
TOASO	x_{21}	3,172	233,212	-0,178	4,296	4,518	0,10
TSKB	x_{22}	2,055	171,299	-0,199	2,592	0,812	0,67
TUPRS	x_{23}	1,975	105,238	-0,063	**2,543**	0,561	0,76
ULKER	x_{24}	1,239	143,948	-0,025	3,779	1,523	0,47
VESTL	x_{25}	0,174	263,331	1,563	9,732	137,757	0,00
YKBNK	x_{26}	2,207	170,699	0,429	3,857	3,672	0,16

*J-B** represents Jarque-Bera Statistic: $n/6\left(\text{Skewness}^2 + (\text{Kurtosis}-3)^2/4\right)$. If the *p-value* is less than 0.05, the null hypothesis of normality cannot be supported at the %5 significance level. Values in bold font signify the highest value for mean and skewness and the lowest value for variance and kurtosis.

Table 2 list the mean, the standard deviation and the coefficient of variation of rate of return of the each stock in ISE-30 index. Coefficient of variation shows the risk per unit return. The ranking of coefficient of variation may provide some preliminary information, with regard to potential candidacy for inclusion in the optimal portfolio. Ranking of C.V. reveals that THYAO ranks at the top of the list, providing the least risk per unit of return, whereas HURGZ ranks at the bottom of list , providing the highest risk per unit of return.

Furthermore, if we consider the coefficient of variation as a risk measure, it can be failed to capture fully the true risk of distribution of the stock return. In this case, the role of higher moments becomes important because true risk should be a multidimensional concept.

Table 2. Coefficient of variation rankings of stocks

Stock	*Mean*	*Std.Dev.*	*C.V.**	*Rank*	*Stock*	*Mean*	*Std.Dev.*	*C.V.**	*Rank*
AKBNK	1,882	13,77	7,32	15	**PETKM**	1,353	11,724	8,67	18
ARCLK	1,563	15,04	9,62	19	**PTOFS**	1,862	13,489	7,24	14
DENİZ	3,733	24,692	6,61	12	**SAHOL**	1,758	14,238	8,1	16
DOAS	2,661	16,476	6,19	9	**SISE**	1,404	11,851	8,44	17
DOHOL	0,739	13,052	17,66	22	**SKBNK**	2,697	17,945	6,65	13
DYHOL	0,452	19,062	42,17	24	**TCELL**	1,558	9,988	6,41	11
EREGL	2,379	12,758	5,36	4	**THYAO**	3,114	13,07	4,2	1
FINBN	1,745	10,12	5,8	7	**TOASO**	3,172	15,271	4,81	2
GARAN	2,682	14,446	5,39	5	**TSKB**	2,055	13,088	6,37	10
HURGZ	0,014	16,621	1187,2	26	**TUPRS**	1,975	10,259	5,19	3
ISCTR	0,98	11,91	12,15	21	**ULKER**	1,239	11,998	9,68	20
ISGYO	0,657	12,186	18,55	23	**VESTL**	0,174	16,227	93,26	25
KCHOL	2,369	12,95	5,47	6	**YKBNK**	2,207	13,065	5,92	8

**C.V.* represents Coefficient of Variation: Mean/Standard Deviation.

Subsequently, in accordance with the second stage, the aspired levels are calculated solving each subproblems by using linear and nonlinear programming:

Table 3. The aspired levels of four moments

	*M**	*V**	*S**	*K**
Objective	3,733	148,86	1,184	0,051

By substituting these aspired levels in (P2), we solve our problem with proposed algorithm. Certainly, the investor preferences not only change in the process, but also affect the portfolio selection. In order to verify the sensitivity of the proposed approach to changes in the investor's preference $(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$, twelve different levels of preference are investigated including also the cases (1,1,0,0), (1,1,1,0), i.e., mean-variance and mean-variance-skewness, respectively. The optimal variables and objective values which are corresponding to the different combinations of $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ are shown in the following table:

Table 4. Optimal value of objectives and trade-off between the four moments

	A	B	C	D	E	F	G	H	I	J	K	L
λ_1	3	3	3	1	1	1	1	3	2	1	1	1
λ_1	1	1	1	3	1	3	2	1	3	1	1	1
λ_1	1	2	3	1	1	1	3	2	3	0	1	1
λ_1	0	1	1	1	3	3	2	3	1	0	0	1
M	2,957	1,714	1,71	1,728	1,721	1,713	1,713	1,713	1,718	3,277	3,15	1,728
V	148,81	148,72	148,33	149,53	148,85	148,85	149,19	148,85	148,85	148,92	148,83	149,53
S	1,18	0,051	0,053	0,049	0,048	0,048	0,048	0,048	0,053	1,187	-0,08	0,049
K	5,288	0,254	0,254	0,254	0,253	0,253	0,253	0,253	0,254	9,757	3,878	0,254
Obj	1,009	5,013	4,973	5,436	62,48	62,48	16,92	62,06	5,107	1,122	2,156	5,436

Investor determines his/her preferences for objective functions with respect to the targeted risk per unit return. As indicated in (P2), the smaller objective function, the better solution is. Thus, investor can select the best portfolio according to the minimal objective functions. But investor should remember the trade-off between objectives since greater preference on return may cause greater risk. As reported in Table 4, the mean-variance efficient portfolio has the highest expected return. This result is consistent is with the notion that the expected return of mean-variance efficient portfolio must dominate any other portfolios given the same level of variance. On the other hand, if the investor chooses the mean-variance efficient Portfolio J, then he/she is exposed to the highest probability of extreme events. To avoid this case, kurtosis can not be neglected as measure of risk. On the other hand, the minimum kurtosis is achieved in Portfolio E, F and H, but objective values of these portfolios are very high. Interestingly, if investor prefers lower preference for variance in Portfolio L rather than Portfolio D, then the same portfolio including also optimal weights of the stocks is obtained. The skewness is only negative in the mean-variance-skewness case. Compared Portfolio B where expected return and variance set equal to those of Portfolio A, higher preference for skewness leads to lower portfolio skewness but also lower portfolio kurtosis than Portfolio A. Similarly, we also consider changing the preference parameters of Portfolio E from (1,1,1,3) to Portfolio A (3,1,1,0) while holding the values of variance and skewness constant. As preference for expected returns increases, the investor must settle for higher kurtosis.

As can be seen, the expected return, variance, skewness and kurtosis are conflicting objectives in portfolio selection problem. That is, as a result of the trade-off between the four moments, at least one of the other three moment statistics deteriorates. Consequently, there is strong evidence which shows that the incorporation of the skewness and kurtosis into the investor's portfolio decision causes a major change in the construction of optimal portfolio since different combinations of investor's preferences on four moments lead to optimal portfolios with substantially different moment characteristics.

Table 5 presents the optimal weights of stocks which should be included in the portfolio. Accordingly, the corresponding weight sets of different risk preference level yield the optimal investment portfolio. For example, for the case of risk preference level (1,1,1,1), the optimal proportion of 26 different stocks is vector (0,052 0,058 0,000 0,075 0,033 0,065 0,000 0,000 0,079 0,067 0,064 0,023 0,071 0,000 0,000 0,067 0,055 0,055 0,000 0,002 0,062 0,072 0,000 0,017 0,037 0,048). Interestingly, FINBN, PETKM, PTOFS, TCELL and TUPRS are not included in any efficient portfolio. Although TUPRS has high ranking of coefficient, the exculison can be the evidence of the importance of higher moments. On the other hand, the lowest ranking stock HURGZ has dominant components except three cases. DENİZ has the most dominant components of 29 percent in mean-variance efficient frontier and it dos not get involded in any model with preference for kurtosis since DENİZ has the highest value of kurtosis. The least preferred stock is ISGYO with the weigth of 2 percent.

Table 5. Optimal portfolio's weights with different preferences of investor's

		A	B	C	D	E	F	G	H	I	J	K	L
	λ_1	3	3	3	1	1	1	1	3	2	1	1	1
	λ_2	1	1	1	3	1	3	2	1	3	1	1	1
	λ_3	1	2	3	1	1	1	3	2	3	0	1	1
	λ_4	0	1	1	1	3	3	2	3	1	0	0	1
AKBNK	x_1	0,06	0,05	0,05	0,052	0,051	0,051	0,051	0,05	0,053	_	_	0,052
ARCLK	x_2	_	0,058	0,057	0,058	0,058	0,058	0,058	0,058	0,057	_	_	0,058
DENİZ	x_3	0,210	_	_	_	_	_	_	_	_	0,29	0,13	_
DOAS	x_4	0,07	0,07	0,07	0,08	0,08	0,08	0,08	0,08	0,07	_	0,072	0,08
DOHOL	x_5	_	0,035	0,035	0,033	0,034	0,034	0,034	0,034	0,035	_	_	0,033
DYHOL	x_6	_	0,065	0,07	0,065	0,065	0,065	0,065	0,065	0,065	_	_	0,065
EREGL	x_7	_	_	_	_	_	_	_	_	_	0,037	_	_
FINBN	x_8	_	_	_	_	_	_	_	_	_	_	_	_
GARAN	x_9	0,180	0,08	0,08	0,079	0,078	0,078	0,078	0,078	0,080	_	0,029	0,08
HURGZ	x_{10}	_	0,067	0,067	0,067	0,068	0,068	0,068	0,068	0,067	_	_	0,067
ISCTR	x_{11}	_	0,065	0,066	0,064	0,064	0,064	0,065	0,064	0,066	_	_	0,064
ISGYO	x_{12}	_	0,023	0,023	0,023	0,025	0,025	0,025	0,025	0,023	_	_	0,023
KCHOL	x_{13}	0,08	0,07	0,070	0,071	0,070	0,070	0,070	0,070	0,070	_	_	0,071
PETKM	x_{14}	_	_	_	_	_	_	_	_	_	_	_	_
PTOFS	x_{15}	_	_	_	_	_	_	_	_	_	_	_	_
SAHOL	x_{16}	_	0,067	0,068	0,067	0,066	0,066	0,066	0,066	0,068	_	_	0,067
SISE	x_{17}	_	0,054	0,053	0,055	0,056	0,056	0,056	0,056	0,053	_	_	0,055
SKBNK	x_{18}	0,045	0,055	0,054	0,055	0,055	0,055	0,055	0,055	0,054	_	0,049	0,055
TCELL	x_{19}	_	_	_	_	_	_	_	_	_	_	_	_
THYAO	x_{20}	0,17	_	_	0,002	_	_	_	_	0,002	0,476	0,307	0,002
TOASO	x_{21}	0,15	0,06	0,061	0,062	0,061	0,061	0,061	0,061	0,061	0,197	0,417	0,062
TSKB	x_{22}	_	0,071	0,071	0,072	0,072	0,072	0,072	0,072	0,071	_	_	0,072
TUPRS	x_{23}	_	_	_	_	_	_	_	_	_	_	_	_
ULKER	x_{24}	0,000	0,016	0,016	0,017	0,018	0,018	0,017	0,018	0,015	_	_	0,017
VESTL	x_{25}	_	0,038	0,038	0,037	0,037	0,037	0,038	0,037	0,038	_	_	0,037
YKBNK	x_{26}	0,040	0,049	0,049	0,048	0,047	0,047	0,047	0,047	0,049	_	_	0,048

5. CONCLUSIONS

This study proposes a Polynomial Goal Programming approach to the mean-variance-skewness-kurtosis based portfolio optimization model. Through the use of the PGP model, an investor can construct a portfolio which matches his or her risk preference based on trading strategies as well as the mean-variance-skewness-kurtosis objectives simultaneously. We illustrate an example in Turkish Stock Market to test our proposed approach with twenty-six stocks from Istanbul Stock Exchange-30 Index. The empirical results indicate that the incorporation of the skewness and kurtosis into the investor's portfolio decision causes a major change in the construction of optimal portfolio since different combinations of investor's preferences on four moments lead to optimal portfolios with substantially different moment characteristics and this confirms our argument that higher moments can not be neglected in the portfolio selection.

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Appendix

Table 6. The Variance-Covariance (σ ij) Matrix

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}
AKBNK	**189,61**	112,52	143,1	160,21	96,374	166,45	60,872	57,013	159,78	147,64	135,71	94,599	135,71	53,701	73,131	150,59	111,77	119,18	58,128	94,849	113,76	116,68	67,023	83,778	109,649	138,894
ARCLK		**226,19**	37,715	155,72	106,04	161,08	88,928	41,794	157,85	150,9	113,92	111,4	139,66	77,339	90,545	156,4	117,52	192,4	42,036	102,85	175,94	137,12	99,662	125,64	187,37	117,2
DENİZ			**609,7**	114,6	73,65	195,37	-14,59	139,93	109,23	163,25	102,16	109,28	102,57	16,37	82,349	99,743	69,951	49,91	38,276	59,492	91,354	84,612	44,703	52,251	-16,573	96,055
DOAS				**271,47**	127,16	204,92	77,326	64,861	187,74	180,89	141,89	125,69	157,98	91,024	115,49	178,84	151,59	186,02	82,751	124,23	181,91	162,86	87,198	124,07	145,85	146,613
DOHOL					**170,36**	193,14	73,844	26,906	108,47	143,46	89,095	83,071	107,98	63,321	74,204	104,36	94,252	122,45	44,578	76,012	115,5	96,37	62,878	77,897	113,676	76,589
DYHOL						**363,36**	79,292	73,144	189,58	277,14	159,19	119,08	167,07	95,689	120,21	164,65	135,61	177,22	61,51	122,64	188,91	146,38	113,13	112,08	149,7	130,352
EREGL							**162,76**	3,667	90,637	57,738	65,236	54,729	88,16	49,093	67,411	85,168	74,767	116,44	14,003	50,797	94,954	78,67	66,526	68,546	118,691	58,277
FINBN								**102,41**	63,464	71,787	50,249	59,717	48,894	29,064	35,046	60,304	37,576	33,387	43,852	30,886	76,727	46,744	28,822	21,163	26,367	56,312
GARAN									**208,7**	167,87	148,33	114,81	157,74	95,283	80,556	174,68	127,62	185,09	67,337	103,2	151,3	155,93	95,844	105,85	152,076	151,579
HURGZ										**276,26**	146,94	126,52	149,49	83,47	103,51	150,03	128,68	158,4	58,591	129,96	159,76	141,28	106,49	112,73	142,144	115,314
ISCTR											**141,84**	103,96	132,15	78,207	78,175	133,63	113,71	122,17	49,798	90,416	111,32	130,77	71,345	88,071	131,702	125,683
ISGYO												**148,5**	110,03	71,402	91,302	101,41	98,479	121,63	51,448	88,713	113,41	112,8	64,764	87,127	127,17	95,78
KCHOL													**167,71**	79,399	91,836	161,9	118,29	158,27	55,238	82,039	135,57	137,3	85,025	105,2	133,222	133,937
PETKM														**137,46**	50,48	75,56	75,856	87,112	56,202	65,132	86,17	82,93	53,468	62,427	85,795	83,736
PTOFS															**181,95**	88,639	84,652	136,94	23,39	61,137	97,779	72,848	47,726	104,1	134,894	73,47
SAHOL																**202,71**	120,63	173,9	58,894	85,71	158,3	146,37	78,964	107,63	144,898	148,583
SISE																	**140,44**	134,29	61,064	103,26	121,93	125,31	68,212	98,273	132,26	98,688
SKBNK																		**322,03**	43,126	107,65	182,16	145,21	107,89	158,51	202,623	124,305
TCELL																			**99,753**	60,141	57,309	56,221	23,93	23,172	29,202	60,994
THYAO																				**170,82**	106,35	115,03	53,065	80,018	105,758	70,747
TOASO																					**233,21**	124,05	105,16	108,74	154,439	108,3
TSKB																						**171,3**	72,491	97,923	145,883	127,959
TUPRS																							**105,24**	58,258	86,581	59,441
ULKER																								**143,95**	141,113	76,061
VESTL																									**263,331**	108,567
YKBNK																										**170,699**

Table 7. Skewness-Coskewness (s_{ijj}) Matrix

	x_1^2	x_2^2	x_3^2	x_4^2	x_5^2	x_6^2	x_7^2	x_8^2	x_9^2	x_{10}^2	x_{11}^2	x_{12}^2	x_{13}^2
AKBNK	**2105,7**	-105,9	5883,7	1001,5	371,2	1228,5	-597,1	1752,8	1534,6	1366,7	1382,2	1053,1	1355,8
ARCLK	127,7	**1353,6**	-2159,0	-252,9	102,1	-775,2	54,5	566,2	846,5	-547,8	180,9	251,5	123,2
DENİZ	16578,0	-706,7	**52668,8**	10424,8	4189,0	15430,5	-3189,6	13652,4	10670,6	13778,0	10544,2	7580,6	10612,0
DOAS	-86,5	-984,3	1600,7	**-1561,1**	-533,1	-954,8	-856,6	609,9	-391,0	-848,1	-619,0	-973,5	-618,4
DOHOL	-33,6	169,7	-166,4	84,7	**1231,5**	1294,9	-276,1	244,4	-191,5	934,5	69,4	97,1	-4,4
DYHOL	606,1	-863,1	2923,5	753,2	2382,5	**3669,5**	-146,5	1241,2	873,7	2519,4	972,1	133,8	991,3
EREGL	-827,5	-953,6	-1430,0	-1432,1	-481,7	-1216,1	**-480,8**	-396,4	-778,8	-867,5	-524,9	-582,6	-843,3
FINBN	1491,7	516,8	3968,4	982,3	599,5	1195,3	110,4	**1200,2**	1192,9	1103,5	851,0	837,3	1356,2
GARAN	1286,5	583,9	1746,4	563,0	214,9	710,9	46,7	1132,6	**1191,1**	501,9	757,6	419,1	676,3
HURGZ	847,2	-315,7	3761,0	-147,8	1582,2	1954,1	47,5	1110,2	417,9	**1945,4**	852,3	595,7	612,6
ISCTR	914,6	-9,9	1876,3	143,7	365,3	844,4	157,9	702,6	633,9	730,5	**674,9**	348,6	594,5
ISGYO	-58,9	-635,5	564,8	-919,1	-131,7	-457,3	-701,7	614,3	-397,8	-170,7	-150,7	**-158,1**	-361,4
KCHOL	716,6	-282,4	2243,6	-34,9	-65,6	466,9	-671,7	1367,3	403,1	384,0	423,8	121,6	**244,8**
PETKM	-796,3	-689,2	-843,0	-999,0	-172,1	-629,3	-14,8	-636,9	-863,9	-677,6	-638,6	-493,9	-355,8
PTOFS	-402,4	-756,9	-920,2	-765,3	-130,1	-904,7	-981,9	312,9	-778,6	-659,1	-544,9	-607,8	-910,9
SAHOL	1134,4	1127,6	2561,2	798,8	222,3	586,7	-251,5	1614,2	1503,1	247,8	820,0	645,9	889,2
SISE	-215,9	-488,0	63,1	-685,9	-121,1	-476,7	-183,4	158,3	-390,6	-262,6	-179,2	-219,3	-296,3
SKBNK	-72,1	1656,1	-2925,5	-249,0	508,7	-596,5	511,2	427,8	840,3	-1252,5	-67,2	-396,4	-307,8
TCELL	-310,9	-629,1	1431,2	-717,0	-168,8	-438,9	-30,8	57,6	-459,2	-451,9	-388,7	-283,2	-291,7
THYAO	-628,9	-738,0	-269,0	-798,0	-262,2	-683,3	-11,5	88,2	-529,8	-485,5	-689,4	-840,8	-823,4
TOASO	-576,0	-472,8	-769,0	-842,2	473,9	-757,1	-730,0	238,5	105,5	-1192,9	-269,6	-266,8	-168,9
TSKB	8,2	-114,5	-136,9	-478,8	-131,6	-185,5	-260,4	537,6	-138,4	-265,3	-188,3	-213,1	-317,8
TUPRS	-136,6	-260,1	-297,1	-237,9	218,9	12,9	-490,3	-61,7	-42,6	-48,6	16,3	78,1	-155,1
ULKER	-298,0	-154,9	-726,6	-935,9	-269,0	-708,9	-42,6	-5,3	-433,7	-801,0	-285,7	-499,1	-654,9
VESTL	1878,3	3496,3	-3629,1	1580,5	1195,3	1274,9	1480,2	178,9	2747,0	1000,3	2082,3	782,0	1242,8
YKBNK	1088,7	-123,1	1927,0	371,6	-162,6	15,7	-221,1	1097,6	856,0	165,0	431,8	25,7	641,7

	x_{14}^2	x_{15}^2	x_{16}^2	x_{17}^2	x_{18}^2	x_{19}^2	x_{20}^2	x_{21}^2	x_{22}^2	x_{23}^2	x_{24}^2	x_{25}^2	x_{26}^2
AKBNK	-52,8	742,4	1461,7	421,6	-119,5	548,6	-23,7	181,1	672,0	136,6	131,2	-158891	1499806
ARCLK	-273,5	-233,3	1032,3	-69,1	1545,3	-190,2	-412,9	20,0	606,2	-378,7	185,0	1859679	346937
DENİZ	484,0	5361,7	11590,5	6080,7	-1000,6	6106,7	3944,0	3550,4	6612,3	1959,8	1542,9	-5688248	10169582
DOAS	-1244,0	-1066,6	-287,8	-1102,0	-762,9	-497,3	-1202,6	-963,5	-778,2	-597,2	-1227,7	-1102280	-366718
DOHOL	56,0	77,9	-119,5	99,1	28,6	-67,2	510,7	776,8	79,9	165,1	-28,4	82454	-312701
DYHOL	993,2	-542,6	632,4	63,6	-653,6	589,0	1381,1	741,6	1022,4	757,1	-515,8	-849250	159215
EREGL	-682,2	-635,7	-774,9	-853,9	-564,2	-662,6	-610,2	-1256,9	-733,2	-806,7	-582,9	-393582	-1011904
FINBN	-254,0	528,2	1436,2	514,6	539,9	497,3	212,7	456,2	730,3	319,7	326,7	-375087	1174896
GARAN	-377,1	416,5	1354,3	154,5	628,4	-32,9	-284,2	676,3	390,4	33,2	68,8	831397	1006894
HURGZ	-83,8	-275,3	142,1	234,0	-763,1	337,4	603,2	-110,9	331,0	437,2	-340,1	-486679	-72317
ISCTR	-212,1	280,2	633,3	209,7	160,8	-9,2	-84,7	258,3	215,7	225,6	23,9	325765	388372
ISGYO	-586,7	-339,9	-417,9	-394,1	-705,0	-151,0	-617,4	-798,4	-491,0	-357,2	-579,7	-617635	-543798
KCHOL	-176,5	-179,5	478,6	-128,6	-682,6	117,4	-332,0	31,6	-24,7	-245,5	-515,1	-588459	427186
PETKM	**-123,7**	46,1	-541,7	-856,7	-138,8	-719,8	-872,2	-638,2	-856,0	-373,9	-266,2	-704247	-622715
PTOFS	-488,7	**-411,3**	-454,2	-360,5	-588,0	-90,1	-793,4	-656,8	-1078,7	-675,3	-381,6	-324707	-953864
SAHOL	-23,8	454,2	**1680,3**	309,7	1020,3	226,1	-304,7	1088,5	847,3	-135,5	135,4	1074505	1252633
SISE	-738,8	-47,6	-262,8	**-270,5**	-329,4	-453,3	-565,9	-412,5	-375,2	-367,2	-281,2	-30995	-576602
SKBNK	73,5	-40,0	1246,0	31,0	**2337,8**	-439,9	-876,8	1104,7	115,0	-118,9	29,7	2774641	515328
TCELL	-611,3	-216,2	-320,4	-497,8	-536,4	**-210,3**	-580,0	-642,6	-465,8	-331,3	-404,1	-739083	-480249
THYAO	-528,6	-618,3	-823,4	-776,2	-691,4	-557,1	**89,7**	-500,4	-540,4	-335,8	-464,1	-611378	-1015020
TOASO	-329,4	-1470,8	552,3	-589,2	417,2	-53,9	-603,1	**-634,4**	575,3	-985,7	-916,8	-207369	-66349
TSKB	-705,0	-319,8	30,0	-470,3	-154,3	-238,4	-565,5	252,2	**-445,9**	-92,6	-456,5	-22524	-314575
TUPRS	52,9	-398,2	-277,5	-178,4	-99,4	77,3	73,7	-563,4	123,3	**-68,0**	-278,2	-197909	-170855
ULKER	-234,4	-98,4	-435,6	-304,9	-234,8	-418,5	-476,4	-539,1	-369,0	-406,3	**-43,4**	472114	-264868
VESTL	190,5	1902,7	2928,6	1782,9	4746,5	-905,2	945,9	2028,7	1947,3	297,4	2520,9	**6681142,0**	1125447
YKBNK	-363,2	-83,6	1017,2	-260,6	235,8	-48,0	-891,2	-196,7	4,5	-296,2	-167,5	-475158	**955947,0**

Table 8. The Kurtosis-Cokurtosis (k_{ijjj}) Matrix

	x_1^3	x_2^3	x_3^3	x_4^3	x_5^3	x_6^3	x_7^3	x_8^3	x_9^3	x_{10}^3	x_{11}^3	x_{12}^3	x_{13}^3
AKBNK	175031,6	100659,3	2371638,0	130521,2	38031,0	193380,9	31899,0	74720,3	112081,8	142980,2	66385,3	47689,4	88532,2
ARCLK	54594,0	219346,3	128245,3	122454,0	46270,5	167826,1	48159,8	20838,5	97996,5	132868,8	38057,8	67172,5	66142,3
DENİZ	338200,8	24123,1	6946561,5	122857,7	22306,7	310784,2	16453,5	188866,4	113124,1	261298,8	96959,1	81059,8	139218,9
DOAS	138645,1	139971,9	1641864,5	205570,8	63933,7	260639,7	43007,5	63126,9	123912,0	157561,0	62853,6	60746,8	89851,5
DOHOL	66383,8	88512,7	724081,0	93478,1	106097,8	299175,5	41254,6	24641,7	63939,8	134703,9	34371,7	47408,1	52673,2
DYHOL	158828,3	137535,8	2303542,6	160640,1	121146,7	554058,2	47102,5	75828,7	112294,9	273025,9	73833,6	77753,9	97620,9
EREGL	11366,3	103077,8	-241237,9	64316,3	32962,2	74119,2	72322,5	-7990,5	50701,4	37023,2	18614,7	39652,4	38422,4
FINBN	106426,9	40806,2	1933863,3	52418,9	7390,8	107708,7	-792,0	76004,4	49835,3	95052,6	33944,4	29313,6	46410,5
GARAN	135470,2	141125,4	1646515,4	152297,2	43398,8	212360,8	45055,6	62068,1	131191,6	135168,0	63477,6	49885,8	88411,4
HURGZ	145270,6	126312,5	2099014,5	134922,4	83246,3	403761,0	36019,1	69438,5	91888,6	267047,3	64760,0	81756,6	84289,8
ISCTR	119541,1	98598,6	1572875,6	119634,1	42630,0	202119,6	32179,2	53331,1	94580,7	140277,6	58475,0	51787,6	73373,0
ISGYO	93528,8	110944,4	1294480,6	113198,4	35312,8	132524,4	32372,9	49529,2	73777,2	133896,8	46425,1	78213,1	67068,2
KCHOL	125441,4	124229,2	1673475,7	128820,3	54083,2	225217,7	47956,0	57292,5	103586,4	145814,7	57423,8	58389,0	93473,3
PETKM	36359,8	42185,1	211755,9	80751,3	38255,4	150671,6	18237,0	18966,0	51785,8	64727,9	27201,9	24431,3	35384,0
PTOFS	74217,9	125538,9	987433,7	101122,2	37453,0	128294,6	45121,6	31326,6	71183,3	116239,9	41003,4	65785,9	63382,9
SAHOL	133012,9	155530,7	1691250,6	148330,0	45318,7	209548,7	44892,4	62740,3	122015,1	138417,0	58628,7	50807,5	92481,0
SISE	90139,6	102452,4	1001249,8	122734,5	46705,0	143989,0	39464,1	39723,2	85530,7	109928,2	49432,4	50356,9	64172,6
SKBNK	58559,6	211330,8	92890,7	137859,0	54747,7	164874,9	65340,8	15443,7	124864,2	125539,7	44795,9	65276,4	72535,4
TCELL	71472,2	24943,8	1038855,2	69939,5	23361,1	101787,5	7390,5	44514,2	45055,0	67455,6	29496,3	28111,9	43342,4
THYAO	69499,3	85291,2	731443,1	103999,8	44559,9	182357,3	26217,2	28056,8	61944,9	133877,8	40413,7	55472,1	50822,0
TOASO	78673,4	173004,0	698950,2	136335,5	59629,0	253336,5	48196,0	43818,0	91582,0	147974,2	42424,5	62651,7	64854,7
TSKB	88835,3	120516,8	1036483,1	128657,9	49418,8	182486,5	39285,9	41498,7	97542,9	124763,6	52404,9	53947,8	73278,1
TUPRS	49961,9	73179,9	459760,5	60888,9	31083,6	150259,9	32771,8	20568,2	43281,0	93532,7	26177,9	35901,5	37046,7
ULKER	54398,7	128112,6	402700,9	104674,1	34720,3	98837,9	41362,1	15359,2	76946,1	94791,3	33437,7	49980,6	53299,0
VESTL	28826,8	221148,1	-525992,0	127191,7	51121,6	111484,7	64095,3	-4449,5	116659,6	100339,3	43325,9	67664,4	57244,9
YKBNK	125444,6	115647,8	1619180,0	126833,4	33866,3	171188,5	31974,3	61407,3	102013,0	113552,9	55245,2	50191,0	86004,8

	x_{14}^3	x_{15}^3	x_{16}^3	x_{17}^3	x_{18}^3	x_{19}^3	x_{20}^3	x_{21}^3	x_{22}^3	x_{23}^3	x_{24}^3	x_{25}^3	x_{26}^3
AKBNK	22469,4	55127,6	117929,2	50295,3	164125,9	28924,1	39525,0	79971,4	54580,4	16873,7	40846,0	236552,6	100449,6
ARCLK	30433,7	88482,0	123705,0	45005,8	312177,9	14492,9	50108,1	185043,8	65564,6	27322,4	74572,0	424015,0	68402,4
DENİZ	5719,7	73234,4	118190,4	37526,1	10949,5	35570,6	28388,1	99496,5	40242,9	10610,4	9949,9	-111285,4	127267,5
DOAS	42621,3	83635,0	136658,9	63376,2	259792,8	31745,6	60750,1	166467,0	75321,9	22909,7	65208,5	316261,5	94886,0
DOHOL	18042,9	49107,2	71429,5	38523,5	145339,7	12122,0	28703,8	98629,0	43630,4	18040,3	32992,9	193085,0	42490,7
DYHOL	41383,7	95309,9	120008,2	57775,1	220010,1	30458,3	53414,6	170627,8	69586,2	30850,2	57029,7	264379,8	89546,8
EREGL	5312,1	57879,4	50901,5	31121,3	141028,6	-5053,4	26391,7	81242,6	41589,0	19740,5	31694,2	247084,7	33067,8
FINBN	14516,1	19792,6	62742,3	16683,2	38382,8	22594,4	17167,2	71332,6	16397,6	6237,4	4778,7	36765,5	55465,2
GARAN	33754,2	58507,3	135552,1	53240,4	270407,8	26483,9	42739,1	131503,1	74098,1	21581,7	58567,8	350173,3	95942,8
HURGZ	34138,8	85389,2	100034,4	46897,6	180344,9	27446,0	55467,7	129084,7	59201,6	26034,1	55580,4	213574,0	80743,1
ISCTR	31779,9	57556,7	93216,1	47605,2	168809,0	22485,6	35243,2	77273,9	58814,1	15988,0	44706,1	247258,6	78141,1
ISGYO	26088,6	82015,8	78413,5	42405,0	157772,1	19679,6	42870,9	83469,8	49281,1	12740,5	42988,7	199304,4	67358,7
KCHOL	28117,2	71297,9	117012,0	49181,8	200628,1	20198,5	34068,2	107624,6	66331,0	20360,9	52105,6	243500,6	91304,8
PETKM	63841,9	19659,4	39289,1	33220,7	100062,2	23847,5	35821,4	42733,4	39087,4	8716,8	27356,5	127128,4	43513,9
PTOFS	14435,6	136983,6	77250,3	40883,7	210434,5	11540,7	37821,9	79863,9	46456,4	14607,8	57145,6	312865,5	58869,1
SAHOL	26189,1	69770,6	156899,5	49010,6	275852,8	22438,1	35150,3	148084,1	68886,8	18082,5	63176,7	359175,6	96599,1
SISE	33436,6	58744,6	85191,0	58011,6	182182,1	24988,2	43589,3	91559,6	59903,3	15907,7	47902,4	250105,4	63004,2
SKBNK	24477,3	113213,5	146467,6	52006,7	452229,1	8577,2	44413,1	189823,0	78984,0	26921,2	100650,3	549847,4	67349,7
TCELL	28854,9	13295,3	43600,1	31264,1	34098,2	33938,1	31887,6	24991,5	25781,3	2192,9	7295,7	16348,8	49045,4
THYAO	24396,5	61757,7	58995,6	44450,6	139866,9	23033,3	78940,1	52836,3	51573,8	9512,1	40998,7	198642,9	46297,5
TOASO	40493,8	85016,5	117483,4	46994,8	261668,1	21831,9	55411,4	233661,5	56003,8	30728,2	60761,5	330267,3	60746,4
TSKB	30932,4	62805,0	103704,1	52333,9	211595,8	19473,1	50697,9	98383,0	76059,0	13776,6	48973,1	279751,0	74728,5
TUPRS	19905,1	38385,8	40376,0	24937,3	96716,3	11335,9	26676,8	94641,1	32417,7	28165,4	24795,8	106892,5	29317,6
ULKER	16951,7	75959,5	87299,5	38625,1	242819,9	6234,5	34851,8	95229,5	51650,1	15320,7	78303,6	326044,0	41452,3
VESTL	25273,1	113749,1	123386,5	59589,2	403736,5	8521,1	48380,0	137109,7	78318,7	21208,2	92982,4	674873,9	55044,9
YKBNK	37306,3	65974,6	114406,1	46691,4	176772,0	26718,3	38959,9	90571,0	61480,6	13539,5	41071,8	231671,7	112375,2