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INVITED ARTICLE

Geocadabra Construction Box: A dynamic geometry interface within a 3D visualization teaching-learning trajectory for elementary learners

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This study focuses on the integration of a 3-D dynamic geometry interface to enhance the 3-D visualization capacity of 8-9-year-old children who attend an after-school program. Each year, all third grade children, who attend a dual-language urban elementary school, are invited to participate, typically beginning with 20-25 participants. The program runs for one hour per week for the duration of the academic year. The research team (a university researcher and one or more classroom teachers) uses design research principles (Cobb, et al., 2003) to develop and refine teaching-learning trajectories for the program. They use socially mediated instructional strategies, constantly challenging learners to find multiple solutions and explanations to a wide variety of spatial problems. Learners work with figures made from wooden cubes, 2-D pictures that resemble these figures, and with iconic representations (such as top-view numeric or top, side and front plane views) that do not directly resemble the figures. Through the integration of Geocadabra (Lecluse, 2005), the 3-D dynamic digital interface, learners move easily among the different representations and then can mentally abstract properties of these figures. They were able to visualize and accurately enumerate cubes of a complex 2-D conventional picture, but were also able to determine multiple solutions for given sets of front, side and top view diagrams, which do not always correlate with only one 3-D solution. With the current curricular focus on predominantly symbolic numeration, systematic integration of visualization, even as a representation tool for number work, into the elementary curriculum is problematic.

Introduction

The National Council of Teachers of Mathematics' Principles and Standards for School Mathematics (NCTM, 2000) recommends that in their early years of schooling, learners should develop visualization skills through hands-on experiences with a variety of geometric objects and use technology to dynamically transform simulations of two- and three-dimensional objects. Later, they should analyze and draw perspective views, count component

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parts, and describe attributes that cannot be seen but can be inferred. Learners need to learn to physically and mentally transform objects in systematic ways as they develop spatial knowledge.

Using design-research principles (Cobb, et al., 2003; Sack, & Vazquez, 2011) this research team has developed a learning trajectory for the spatial development of elementary children using cube structures. This paper focuses on the interchange among specific types of representations, guided by the Spatial Operational Capacity (SOC) framework developed by Van Niekerk (1997) and the critical role of a dynamic computer interface. The project's strong problem-solving approaches make this possible. It is conducted in a dual-language urban elementary school within one of the largest public school districts in the mid-southwestern United States. More than 70% of the school's students are designated "At Risk" and at least 50% of its students are English Language Learners.

Theoretical backdrop

The *Spatial Operational Capacity* (SOC) framework (Van Niekerk, 1997; Sack, & Van Niekerk, 2009) that guides this study exposes learners to activities that require them to act on a variety of physical and mental objects and transformations. While the entire SOC framework encompasses a large range of representations and operations in the field of visual learning, only those pertaining to the work of this project are presented here, shown in Figure 1.

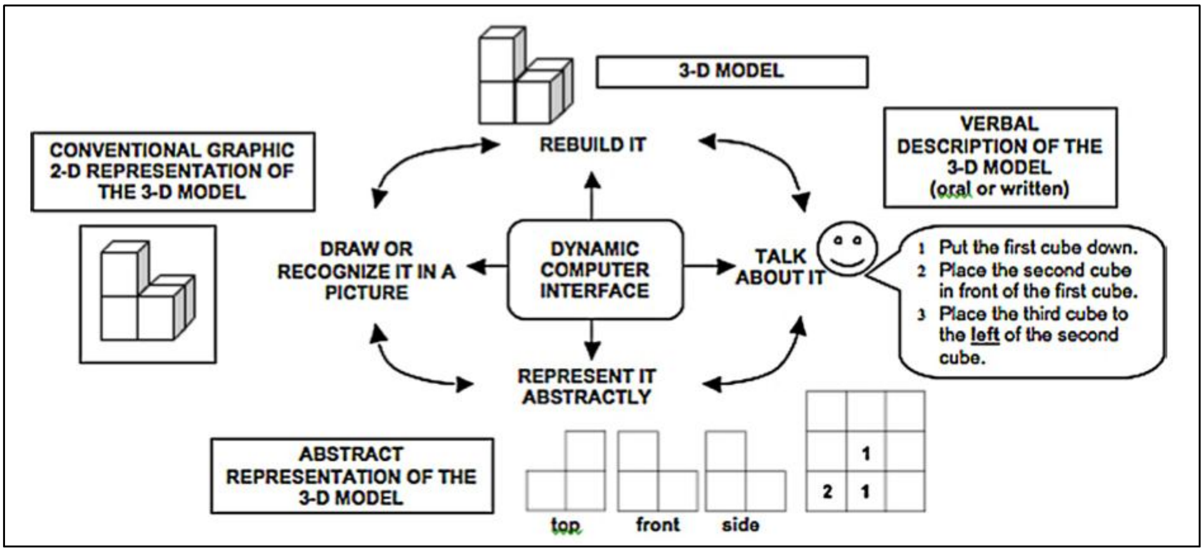


Figure 1: SOC Representations

Within this trajectory, learners interact with full-scale figures, that, in this study, are created from loose cubes or Soma figures, made from 27 unit cubes glued together in different 3- or 4-cube arrangements (see Figure 2); conventional 2D pictures that resemble the 3D figures; verbal descriptions that may be accompanied by gestures using appropriate mathematical language (Sack, & Vazquez, 2008); and, abstract representations such as front, top and side views or numeric top-view codings that do not obviously resemble the 3D figures. Numeric top-view codings are 2D representations that show the number of cubes standing on each cell of the top-view grid. This may be a useful representation for enumerating total cubes in such figures, including rectangular prisms.

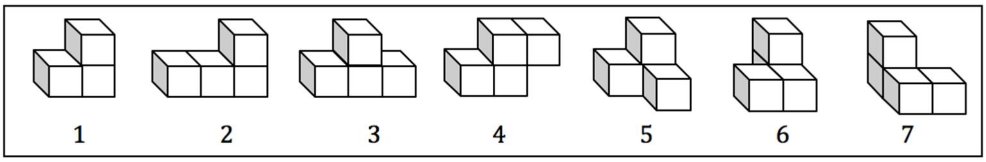


Figure 2. The Soma set can be made by gluing unit cubes together.

The dynamic computer interface, Geocadabra (Lecluse, 2005), through its Construction Box module, allows learners to construct, view and manipulate complex, multi-cube structures as 2-D conventional representations or as top, side and front views or numeric top-view grid codings (see Figure 3). By clicking successively on a grid position on the key pad shown in Figure 3, a corresponding stack of cubes appears. By right-clicking, the stack may be reduced in height or removed. As the figure is constructed, the front, side and top views dynamically change. The show (hide) views, key pad or 2D figure options can be pre-selected according to instructional goals. The Control-line-of-view option allows the user to move the figure dynamically using the mouse or by clicking on the arrows at the ends of the space's triaxial system that appears on the Construction Box control window. The size of the top-view rectangular grid can be adjusted from 2 to 8 units in width and depth according to user preference.

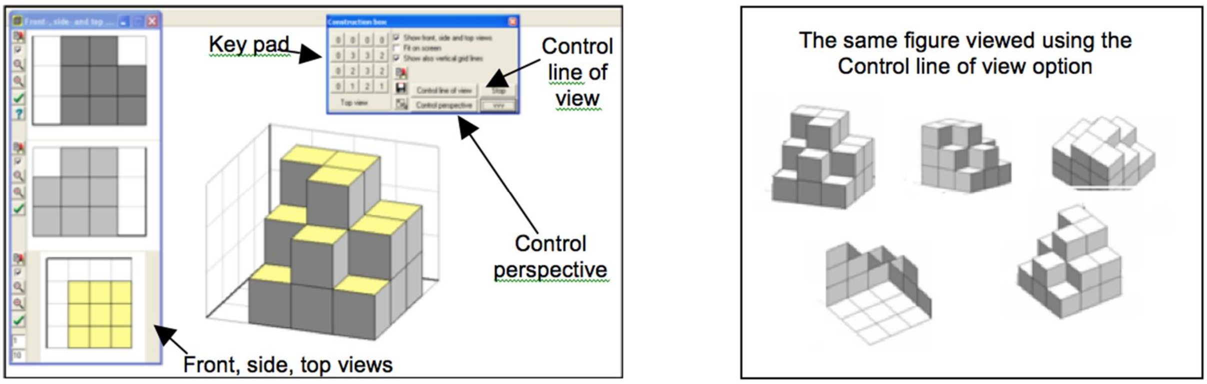


Figure 3: The Geocadabra Construction Box.

Connell's (2001) action-on-objects metaphor is a useful guide for making sense of the place of the Construction Box in the entire framework. Through carefully designed activities, learners act strategically upon manipulative objects as they solve problems. Computer images that replicate the attributes of the physical objects then behave as real objects in the mind of the learner. The Construction Box interface integrates the SOC representations in the form of a dynamic image that can be moved to provide the same views as if moving about a 3-D object; a 2-D image when the figure remains static; and if selected, simultaneous abstract top-view numeric, or face view representations. Follow up problems or extension questions require the child to relate to the newly instantiated and defined object of thought, which becomes the basis upon which later mathematical thinking occurs. This model extends as the child develops his or her own problems based upon the objects that were recently defined. "This ability, to pose one's own problems and to then successfully solve these problems, provides further opportunity for growth in mathematical thinking and problem solving" (Connell, 2001, p. 161).

Context and methods

Since the project's inception in 2007-2008, a university-based researcher and two teacher-researchers have formed the research team working with a group of 3rd-grade (8-9 years old) learners weekly for one hour in teacher-researcher, Vazquez' 3rd-grade classroom within the school's existing after-school program. English and Spanish parent/guardian and student consent-to-participate forms are sent home to parents of all 3rd grade children. All respondents are accepted into the program. The research team uses socially mediated instructional approaches to support a problem-solving environment that fosters learners' creativity according to readiness and interest. The learners are constantly challenged to find multiple solutions to individual problems. They may produce multiple products or multiple solution pathways leading to a unique product.

Design research methodology (Cobb, et al., 2003) guides this research team's instructional decisions based on teaching-learning trajectories developed from an instrumentalist standpoint (Baroody, et al., 2004). This conceptual and problem-solving approach aims for "mastery of basic skills, conceptual learning, and mathematical thinking" using any "relatively efficient and effective procedure as opposed to a predetermined or standard one" (Baroody, et al., 2004, p. 228). This research team's method follows principles of lesson study (Sack, & Vazquez, 2011) in that each lesson is part of a design experiment followed by a retrospective analysis in which the research team determines the actual outcomes and then plans the next lesson. This may be an iteration of the last lesson to improve the outcomes, a rejection of the last lesson if it failed to produce adequate progress toward the desired outcomes, or a change in direction if unexpected, but interesting, outcomes arose that are worthy of more attention. Data corpus consists of formal and informal interviews, video-recordings and transcriptions, field notes, learner products and lesson notes.

The next section details the teaching-learning trajectory, now stabilized in its 6th year. It begins with hands-on activities in purposefully kinesthetic ways to foster the development of relatively simple perception images in learners' minds. Zaporozhets (1982/2002, p. 91-2) states, "The sensory fabric, which has its source in movement, and action, which initially is practical and then is perceptual, play a leading role in the formation of a spatial image." These initial activities, moving between 3D and 2D conventional pictures within the overarching SOC framework, become increasingly more demanding prior to their introduction to the Construction Box digital interface. At this point in the trajectory, learners are relatively familiar with the figures that they will represent digitally. The digital interface provides them ways to integrate the abstract representations with their 3D models or 2D conventional pictures. Later on, the trajectory moves away from the digital interface with more complex activities that provide these young learners the ability to express clear conceptual understanding of rectangular solids or complex irregular figures made from cubes. This includes the use of numeric top-view coding to determine and to represent the actual volume of complex structures.

The teaching-learning trajectory

Early kinesthetic activities

Using the SOC framework (Figure 1) as a guide, beginning activities provide learners 3D stimuli requiring production of 3D objects. To further develop their 3D perception, later activities include 2D conventional pictures as stimuli and products but still requiring interaction with 3D objects. In the first activity, *4-cube houses* (a contextual situation adapted from van Niekerk, 1996), learners design houses for a remote outer-space village using 2-cm loose wooden cubes. The houses, whose four rooms much touch face-to-face with no edge overlaps, will be pre-fabricated and each house must be different from all others in the village. Thus, learners become familiar with a variety of 4-cube structures, using transformations and symmetry to determine and justify uniqueness. When later presented with a picture strip of the Soma figures (Figure 2), they recognize some of the houses from this early activity. Each learner then checks that his or her set of Soma figures matches those printed the strip, moving between 3D and 2D conventional representations. While Soma #6 and Soma #7 were previously identified as different houses in the first activity, now learners must find a way to identify them each by their number name even when the strip is not available.

During the next two to three sessions, learners are provided with sets of researcher-created 2D task cards. These illustrate a variety of assemblies of two Soma combinations in different orientations. Learners must identify the two Soma figures that together create the figure shown on the task card. In many cases, more than one solution is possible. Examples of these task cards are shown in Figure 4. Challenge varies according to each learner's level of readiness and interest. They can work with task cards that might be color-coded, shaded or unshaded. These activities are more challenging in that learners use 3D structured combination figures rather than loose cubes. Through their different configurations the Soma figures provide a high degree of complexity and constraint to the instructional tasks and force the learners to engage in a variety of mathematical tasks including mental transformations in ways that would not be possible using loose cubes.

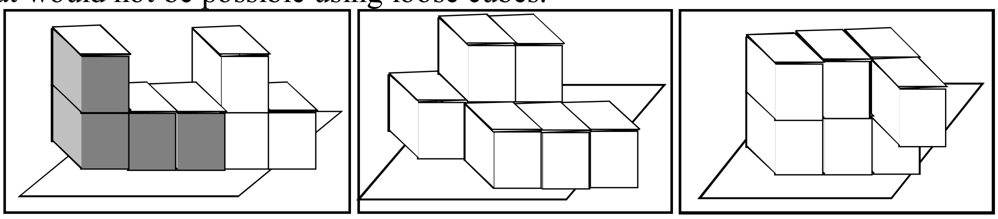


Figure 4:. Two-Soma assembly task cards increasing in difficulty levels

Geocadabra Construction Box activities

By the second month, learners begin to digitally reproduce a variety of figures (e.g., see Figure 5) printed in a customized manual originally created by Van Niekerk and customized by Sack and Vazquez as this trajectory has evolved.

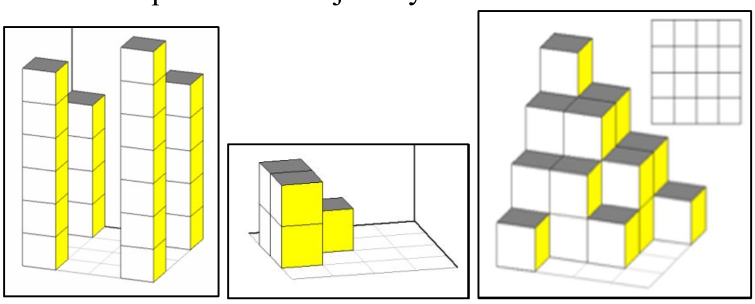
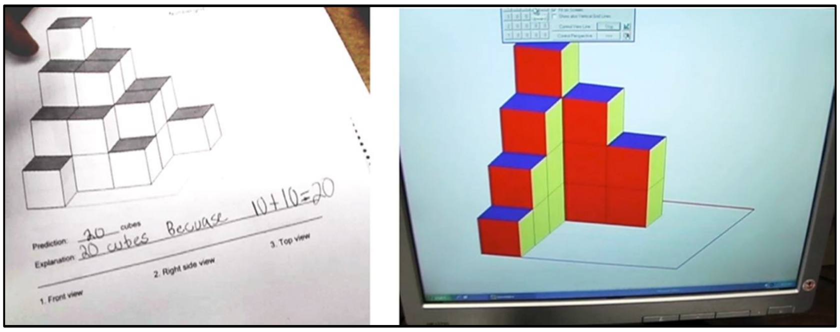


Figure 5: Figures from the Geocadabra manual

These activities provide the learners opportunities to coordinate numeric top-view codings with 2D pictures. They write the numbers in printed grids and later, draw and number the grids themselves. In addition, there is a strong focus on enumeration of cubes in the manual's figures. Whereas beginning learners generally are able to determine the numbers of cubes in relatively simple figures (such as the left-most task card in Figure 4), very few can do so with more complex figures containing hidden cubes (as to the right in Figure 5). Battista (1999) has shown that many learners count visible faces when asked to find the number of unit cubes in 3D rectangular arrays, often double counting edge cubes and triple counting vertex cubes. This research team's pre-program interviews using both 3D rectangular arrays and multi-level irregular structures, such as that to the right in Figure 5, yielded similar results. Some children recognize that cubes are hidden but initially lack the mental structuring capacity to logically determine precisely how many.

The figures in the manual increase in complexity as shown in Figure 5. The research team pays close attention to each learner's ability to accurately enumerate. Sometimes, whole group discussions focus on enumeration, in classes where several learners may still have difficulties perceiving hidden cubes accurately. Volunteer learners share their methods for counting. These include using the 2D digital figure (projected on a large screen for the whole class) or by summing the numbers in the Construction Box grid. Since all have attempted the problem and are familiar with the working of the interface, reconceptualization occurs quite easily for those still struggling with the perception of hidden cubes. In a few cases it has been necessary for some strugglers to rotate the figure on the screen using the Control-line-of-view option to see the hidden sides of the figure.

Figure 6 shows the work of a learner who had not yet been exposed to the whole group discussion described above. Her initial count of 20 cubes from the picture indicated that she was unable to accurately perceive the hidden cubes. The researcher asked her to construct the figure on her computer. She chose to build the left side first and then the back, counting by ones as she tapped the squares on the Construction Box grid.



Field notes, Feb. 8, 2012
[She] predicted that the figure contained 20 cubes. She counted each stack as she constructed it in her own way on the computer and realized that she had missed some hidden cubes at the back.

Figure 6: Finding the missing cubes

After developing reasonable proficiency with the Geocadabra Construction Box through the manual's tasks, more open-ended problems are posed to further develop learners' mental imaging capacity with respect to volume concepts. They create their own structures consisting of 24 unit cubes, using the Geocadabra Construction Box. Initially, they may choose to build the figure using 24 loose cubes, but most discard the loose cubes immediately. The research team converts these learner-created conventional 2D images of these digital figures into new task cards. The creator then draws the numeric top-view code. A peer decodes and re-creates it on the computer, first hiding the visual figure on the screen with the hide/show option, and then showing it to check that it matches the task card figure. In addition, the peer decoder ensures that the figure consists of exactly 24 cubes by adding the numbers in the top-view numeric grid on the computer. Examples of learner-created task cards are shown in Figure 7. Enumeration of these figures using symmetry and slicing is encouraged and shared during whole class discussions when mathematical academic language develops. The research team has noted that learners seem to have more difficulty enumerating 3D rectangular arrays than the types of figures shown in Figure 7.

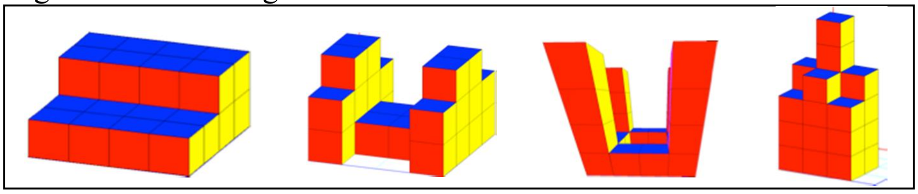
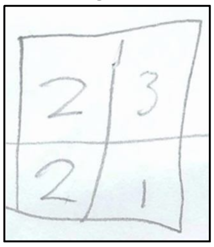


Figure 7: Learner-created 24-cube task card figures.

To reinforce learners' abilities to move among the SOC representations, particularly from the top-view numeric grid as a stimulus, learners used the Construction Box to create more challenging puzzles. Using a complete set of seven Soma figures, they first select two to create a 3D assembly structure that can be reproduced on the Construction Box (unlike the rightmost structure in Fig. 4, which has an overhanging cube). The following week, without the aid of the computer interface, each learner draws the numeric top-view coding from the 2D picture of his or her own structure that the research team has formatted into a task card. Regardless of whether they remember which Soma figures were used they know that the figures were assembled from two different ones since these pictures are their own creations. These grid codings become puzzles for their peers to decode only using the Soma figures. An example is shown in Figure 8.



Solution 1: Soma #7 and Soma #4
Solution 2: Soma #3 and Soma #5
Solution 3: Soma #6 and Soma #2

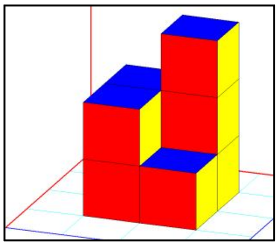


Figure 8: Coding puzzle, three possible solutions, and original task card

Front, side and top views development

During the 2010-2011 academic year, the research team presented activities relating to front, side and top views. Time constraints had precluded this topic for prior groups. The learners were presented with a variety of tasks:

- Given a 2D picture (examples shown in Figure 9), predict and draw the top-view numeric grid; construct the figure on the Construction Box to verify the numeric grid prediction; draw the front, side and top views as shown on the computer screen. Use loose wooden cubes if needed by personal choice.
- Given a 2D picture predict and draw the top-view numeric grid and the front, side and top views; construct the figure on the Construction Box to verify the predictions. Use loose wooden cubes if needed by personal choice.
- Given the front, side and top views (example shown in Figure 10); predict the top view numeric coding; construct the figure on the Construction Box to verify the prediction against the computer-generated figure and its views.

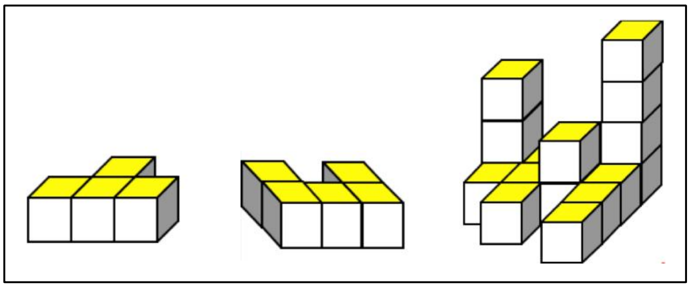


Figure 9: Examples of 2D figures used to create top, side and front views.

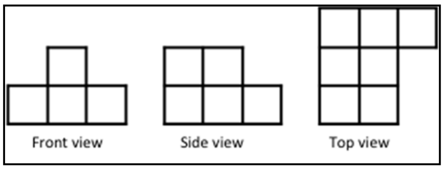


Figure 10. Example of a top, side and front views problem

Learners work at their own paces according to each one's capacity to abstract. Some need to build the given figures on a mat using loose wooden cubes. By rotating the mat the front and side views can easily be seen and correlated with the views generated on the Construction Box. They begin to correlate the outline of the top view numeric grid with the top view, through whole group discussion, recognizing the redundancy of the zero grid spaces that they had become accustomed to drawing from earlier activities. Some are able to visualize that multiple solutions are possible for certain views problems such as that shown in Figure 11. The research team expected all of the children to identify a 2-by-2-by-2 cube to be a solution. Four additional solutions can be produced if one of the cubes on the upper level is removed. The child whose solutions are shown in Figure 11 actually produced two more solutions in which two cubes are removed diagonally from the upper level. She was able to reason with certainty about why there were exactly seven solutions for this problem. During a subsequent whole-class discussion, classmates viewed a video-clip of her explanation in which she used the Construction Box to show how the views of the seven different structures did not change.

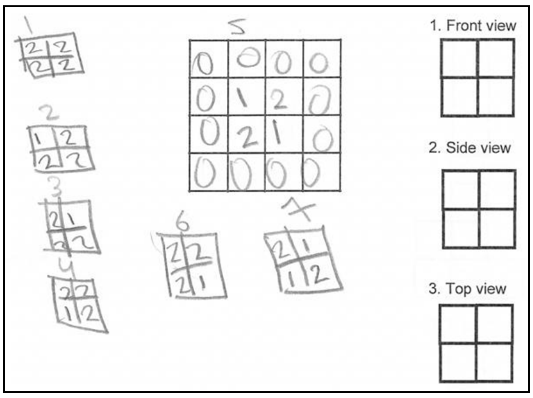


Figure 11: Seven solutions to a views problem

Representing rectangular prisms

During Year 2, an unexpected and interesting event occurred when five Year-1 participants returned. They were struggling with the concept of rectangular volume in their regular academic classes where they were required to use the formula. Using an initial learning trajectory based on the work of Battista (1999), the group attempted to solve volume problems by folding nets drawn on grid paper. They struggled to connect the dimensions of the flaps of each net with the height of its corresponding 3D figure. Within the study's problem-solving environment, using a contextual scenario, "Ms. Moral's Shoes," 24 shoeboxes must be shipped to a nearby city. The learners, using loose wooden cubes to model the shoeboxes, were expected to find all possible combinations of rectangular arrays with 24-cubic-unit volumes. The research team was surprised to see them record their findings as numeric top-view codings rather than directly with the length-width-height formula. Connections between top-view coding and the volume formula evolved through guided discussion among the teacher and participant learners. Figure 12 shows how they recorded their work.

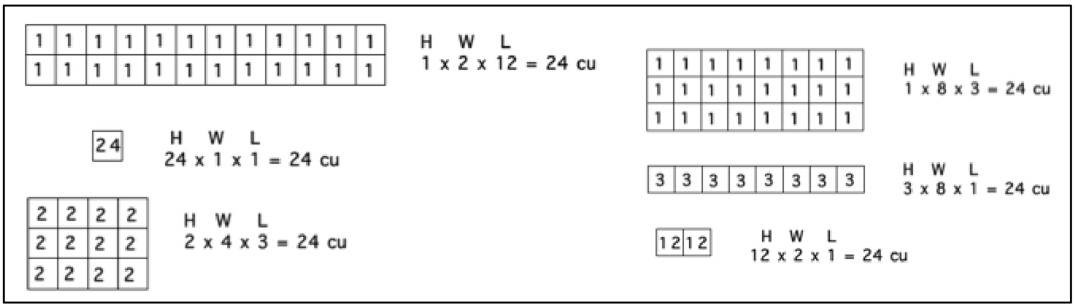


Figure 12: Recordings of 24-cube prisms.

The research team felt that this result was significant and sufficiently important to integrate into the teaching-learning trajectory for future cohorts. Therefore, when satisfied that learners have mastered the ability to move back and forth among the different representations without the Construction Box interface, the class moves away from the computers. The next objective is to find ways to represent rectangular prisms made up of loose cubes. The teacher presents a 12-cube array and invites learners to demonstrate how to code different orientations of this model. Two are shown in Figure 13. Then, they build and code a rectangular array using 24 loose cubes. They are challenged to find as many different combinations and orientations of these 24-cube arrays as possible.

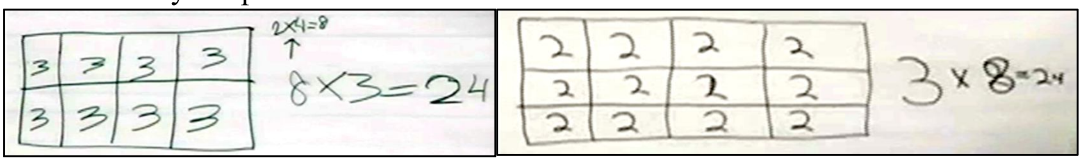


Figure 13:. Learners' representations and enumeration of 3D arrays

Impact of the Construction Box interface

In accordance with NCTM (2000), this project has enabled participant learners to develop appropriate 3D visualization skills. These include the ability to physically and later mentally transform and also represent 3D objects systematically in increasingly abstract ways. Actions on objects (Connell, 2001) occur concretely with the 3D models, virtually through the Geocadabra Construction Box interface and ultimately as mental imaging through the powerful problem-solving approaches developed by the research team. It is remarkable that the children are able to move from the abstract top-view numeric, and front, side and top views representations to the 3D models independently of the computer interface (Connell, M., personal communication, September, 2010).

Outhred et al, (2003), referring to the complexity of dealing with volume measurement compared to rectangular area measurement, state that “the process is more complex because learners have to coordinate three dimensions and diagrams cannot show the layer structure clearly” (p, 84). That these young learners in this study are easily able to transfer their knowledge of top-view coding to an easily drawn representation and to enumeration of 3D arrays is significant. This knowledge is a direct product of their interaction with the Construction Box dynamic interface, which enabled them to coordinate the SOC representations simultaneously.

The research team’s attention to classroom culture is also a significant component that contributes to the success of this project. Learners are encouraged to share their solution processes, sometimes dynamically projecting their Construction Box actions, during whole class discussions. Their listeners are invited to challenge or to add different perspectives during these presentations. Learners also create problem tasks for their peers to solve and check. This means that the levels of complexity are situated where the learners are rather than where the teacher might have anticipated them to be. Since it is unlikely that no two learners will be on exactly the same processing level, this also provides them with additional opportunities to learn while they negotiate, explain and are being challenged by peers.

How this work will be integrated into the regular third grade mathematics curriculum is a problem that this research team has yet to study. Fourth-grade teachers are easily able to identify those children who have completed this program the year before. Their performance on regular mathematics tasks involving visualization, even at the 2D level, surpasses their peers who have not participated in this program. The current climate of teacher accountability through enforced state testing in the US has interfered with delivery of such curricular activities. Therefore, it is unlikely that these will be incorporated into regular classrooms until the focus turns toward deeper and more interesting mathematical exploration than multiple-choice item practice can provide.

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