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INVITED ARTICLE

GeoGebra in Professional Development: The Experience of Rural Inservice Elementary School (K-8) Teachers

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GeoGebra is an emergent open-source Dynamic and Interactive Mathematics Learning Environment (DIMLE) (Martinovic & Karadag, 2012) that invites a modeling perspective in mathematics teaching and learning. In springs of 2010 and 2012, GeoGebra was integrated respectively into two online professional development courses on mathematical problem solving in a rural region of a Midwest state in the USA. Using GeoGebra as a primary tool for modeling and communication, fifty-three K-8 inservice elementary school teachers with no or little previous exposure to DIMLEs participated in the graduate level course, where they experimented with various resources of GeoGebra for mathematical modeling in the broad context of problem solving. The instructional team incorporated a variety of pedagogical components to support teachers' exploration of the new technologies and the mathematical content, including video-based demonstration, affective intervention, learning with children, and social and cognitive scaffoldings. After intensive online instruction, a 25-item questionnaire was administered to collect data on participants' attitudes, curricular awareness, mathematical content, and pedagogical reflection regarding the integration of GeoGebra. Based on results of the questionnaire (N = 45), this article provides a preliminary description of inservice elementary school teachers' learning experience in a GeoGebra-integrated DIMLE setting, including an exploratory dimension analysis of the survey data. Overall, the use of GeoGebra challenged teachers' view about the nature of mathematics and student-teacher interactions and further enriched their mathematical knowledge and pedagogical choices. The course design, context, and implications for future efforts are discussed.

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Introduction

Mathematics teachers' professional development (PD) has become a vital component of the ongoing educational reform in the United States, driven by accountability, diversity, and especially the constant advances of educational technology and innovative instruction paradigms (Darling-Hammond et al., 2008). The need is demanding in rural areas, where classroom teachers are facing both curricular challenges and a lack of innovative resources in many subject areas, particularly in mathematics and science. To meet the professional needs of these full-time classroom teachers, we have in the past four years (2009-2012) implemented a state-funded PD project titled *Science and Mathematics Action Research for Teachers* (SMART), blending face-to-face summer institutes and year-long graduate-level courses in content and pedagogy through online instruction supported by a Course Management System (CMS) and open-source DIMLE technologies (Martinovic & Karadag, 2012). In a mathematics methods course on problem solving, GeoGebra was fully integrated into all instructional components for a variety of instructional functions, including dynamic demonstration, computation, graphing, modeling, exploration, alternative solutions, and online mathematical communication. The first cohort of 27 teachers was first exposed to GeoGebra in Spring 2010 and graduated in December 2010. The second cohort of 26 teachers experimented with GeoGebra in Spring 2012 and graduated in December 2012. Both cohorts were recruited from classroom teachers in the US Midwest and participated in hybrid (online and face-to-face lab days) graduate studies in mathematics and science teaching at a medium-sized state university. In this article, we report on these inservice elementary school teachers' self-perceived experience with GeoGebra-integrated professional learning in mathematical problem solving. Our purpose is to provide an informative description of inservice elementary teachers' learning experience with GeoGebra-integrated mathematical problem solving and further identify the primary themes that characterize such an innovative professional development program.

Theoretical Frameworks

In designing a primarily online course for mathematical problem solving, we were aware of the challenges of both the mathematical content and the limitations and affordances of the delivery platform. Thus, we sought to be guided by both research in mathematical problem solving and theories of instructional design. In mathematical problem solving, we were informed by Schoenfeld's (1985) framework to address the global issues of problem solving—resources, control, heuristics, and beliefs. At the content-specific level, we implemented Polya's (1945/2004, 1981) four-phase strategy for mathematical problem solving, which includes stages for understanding the problem, planning, implementation, and reflection. Both theoretical perspectives were explicitly introduced to the teacher participants through reading and video-based problem solving demonstrations.

As an overarching framework for instructional design, we were guided by Merrill's (2002) First Principles of Instruction, placing emphases on knowledge application and integration. In light of the content requirements of mathematics, we were also informed by Merrill's (2007, 2012) "A-Pebble-in-the-Pond Approach to Instructional Design," which takes a content-oriented whole task as the starting point, gradually progressing to component skills, strategies, interface, and production. In the course work for inservice mathematics teachers, focal ideas of school mathematics were presented as whole tasks, where component technical skills and mathematical ideas interact in meaningful ways in support of the goal of problem solving.

New DIMLE technologies provide a platform to implement the first principles of instruction

(Merrill, 2002, 2012) and bring interactivity and dynamism into teachers' mathematical learning experience (Martinovic & Karadag, 2012). To incorporate modeling and simulations into the online course, we were guided by Model-Centered Learning and Instruction (de Jong & van Joolingen, 2008; Milrad, Spector, & Davidsen, 2003; Seel, 2003), which allows the instructional team to address participants' pre- and/or misconceptions of mathematics using model-specific language and methods and subsequently encourage them to build interactive and dynamic conceptual models as a way to learn, to teach, and to foster metacognition. In trying to represent and solve a mathematical problem using DIMLE resources, participant teachers not only became aware of the problem space and the nature of their understanding, but also used their DIMLE models to explain or justify their conceptions with fellow teachers and their students. We used GeoGebra (www.geogebra.org), an open-source and web-ready DIMLE, as a primary mathematics learning technology (Hohenwarter & Hohenwarter, 2009). With the second cohort (2011-2012), we specifically underlined the role of modeling in mathematics teaching and learning as stipulated in the US Common Core Standards for Mathematics by National Governors Association Center for Best Practices /Council of Chief State School Officers, which were released to the public in June 2010. In light of the comprehensive nature of teacher professional development, the instructional team strived to synthesize domain-specific research findings, theoretical frameworks for technology-integrated instruction design, and emerging policies and standards, as is also required by funding agencies.

Methods

Participants

In the first cohort (2009-2010), twenty-seven inservice elementary school (K-8) teachers were participants in the primarily online graduate course on mathematical problem solving. In the second cohort (2011-2012), twenty-six teachers of comparable experiences were enrolled. The participants were all full-time classroom teachers. At the time of instruction, they were teaching various subjects, including language arts, social studies, math, or science. They were located across the southern region of a Midwest state in the US with a broad span of teaching experiences. The teachers were enrolled in the professional development graduate program, SMART, because of their self-perceived need for professional growth in mathematics and science teaching and the support of their school districts. Virtually nobody reported previous acquaintance with GeoGebra, although a few teachers were aware of the existence of open-source software. Among the 53 participants, forty-five (45) completed the post-course survey about GeoGebra use in their course work.

Instructional Context

GeoGebra was fully integrated into the online course on mathematical problem solving. The course was specifically designed to engage classroom teachers with the processes of problem solving in alignment with state and national standards. The overarching goal is to provide inservice teachers with a holistic perspective on the historical, cultural, and technological aspects of mathematics and the mathematical teaching and learning. Given their teaching experience and daily access to students, the teachers were explicitly encouraged to experiment their newly acquired teaching skills and methods with their classroom students and further reflect on their students' cognitive processes. Within the overall emphasis on teaching for understanding, these teachers were guided to learn with understanding, utilizing the DIMLE resources such as multiple representations, dynamic modeling and simulations, and the social support in the online community.

Instructional Procedures and Interventions

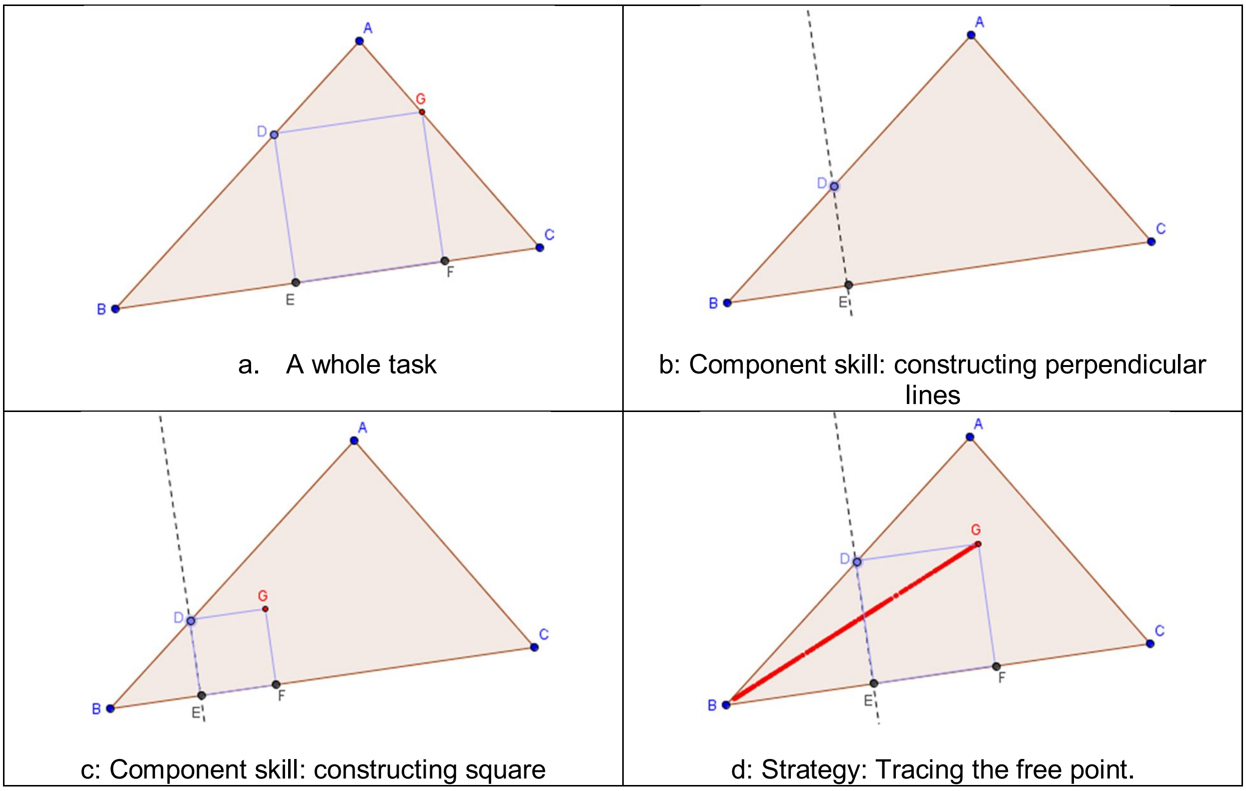
Video-based demonstrations. In teaching classroom teachers about new DIMLE technologies such as GeoGebra, we met many instructional and technical challenges. During the first few weeks with each cohort, we made several instructional videos demonstrating the features of GeoGebra, the processes of problem solving, and their relevance for mathematics teaching. Some teachers were overwhelmed by the combination of the unfamiliar mathematical problems, new ways of thinking, and the use of new DIMLE tools. By contrast, others were excited by the new opportunities. All these initial reactions were within the expectations of the project team and were documented in the discussion forums on the course website. These instructional videos provided an asynchronous form of instructional support and proved to be highly effective for teaching about the features of GeoGebra in an online instructional environment.

Emotional intervention. Our participants have diverse teaching experiences and mathematical content preparation as well as different attitudes and beliefs about mathematics teaching. During the first few weeks with the first cohort, the instructor realized that there was need for an emotional or metacognitive intervention. In a traditional classroom, it is relatively easy to discern such tensions among students. In an online setting, however, such emotional reactions unfolded in two directions. Some participants were blaming their difficulties on the technology; others were overwhelmed and frustrated, as evidenced in the discussion forum. A short survey was thus administered to gather feedback from the participants. The survey results were largely positive, which pointed to the fact that most of the participants, being teachers themselves, were aware of the challenges of learning. Those who felt stressed out by their self-perceived "weaknesses" might have read others' postings online and felt challenged by the "competition". Through emails and online forums, the course instructor sent out messages explaining the emotional aspects of learning and especially mathematical problem solving. Selected sections of Schoenfeld's (1985) and Polya's (1945/2004) work was subsequently given as reading assignments.

Scaffolding problem solving. From a Vygotskian perspective, learners ought to be scaffolded pedagogically so that they could explore a problem situation within their Zone of Proximal Development (ZPD) (Vygotsky, 1978; Wood & Wood, 1996). In an online environment, it is difficult to provide one-on-one scaffolding on a timely basis. To help participants learn about the heuristic strategies for problem solving, the instructor designed interactive web-units, using Quandary® (http://www.halfbakedsoftware.com/quandary.php), to provide instructional support. A short questionnaire followed and revealed that the participants found Quandary-based scaffolding very helpful. Further, team scaffolding is a DIMLE resource that is more flexible than that in traditional instruction. There was always somebody, among the teachers, who could make a good comment or a good "mistake" to push the whole class to move forward. Weekly forums turned out to be a powerful social channel of communication and problem solving, including the discussion of GeoGebra features and their relevance to mathematical problem solving. With the instructor providing occasional guidance, all the problems were addressed in multiple dimensions, some very thoroughly.

Mathematical content. We organized the course content around three major components: theoretical readings, open-source DIMLE technology GeoGebra, and mathematical content, including geometry, algebra, probability and data analysis, and some discrete mathematics. Every week, there was a theme emphasized in mathematical problem solving such as working backwards or visualization. Five to ten mathematical problems were discussed online each week by the whole class and subsequently summarized by each individual teacher.

In selecting and designing learning tasks, we followed the basic framework of Model-Centered Learning and Instruction (e.g., Seel, 2003) and that of the Pebble-in-the-Pond Approach (Merrill, 2007, 2012). A typical instructional task starts with a realistic or real-world problem, whose solution calls for explorative moves by the participants, social interactions, and the use of GeoGebra tools. Component mathematical contents, technical skills, and strategies were discussed online through the weekly discussion forums largely driven by the teacher participants under the guidance of the instructor. While whole-class discussions were documented online, individual teachers would need to complete their own dynamic worksheets for evaluation and reflection. In essence, the instructional team aimed to integrate content, pedagogy, and GeoGebra in a way that allows the three dimensions of teachers' knowledge to grow in synergy (Koehler & Mishra, 2008). Most of the learning tasks were developed under the collaboration of a mathematician with a long-term commitment to teacher professional development and a mathematics educator who is familiar with the instructional affordances of GeoGebra and web technologies. An example that is representative of the nature of the learning tasks is presented in Figure 1, where teachers were asked to inscribe a square within an arbitrary triangle, a classic problem documented by Polya (1945/2004) and Schoenfeld (1985).

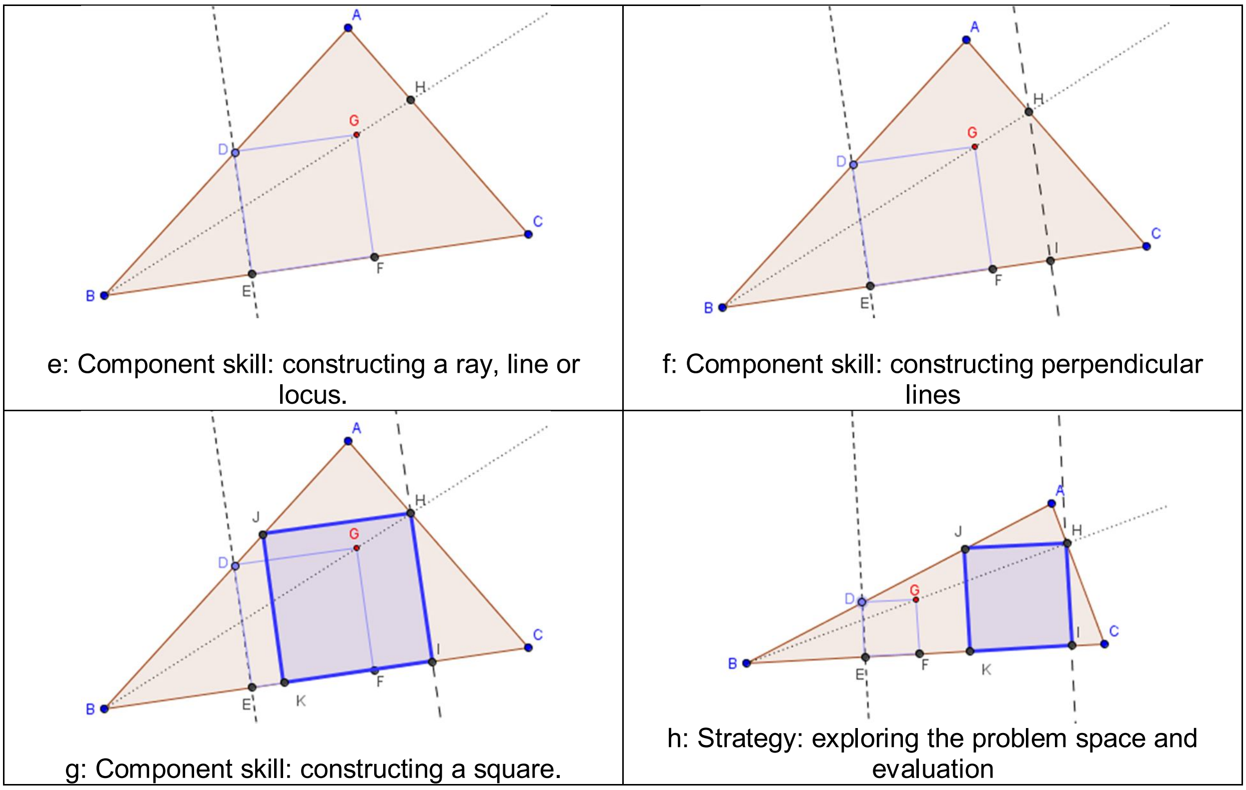


a. A whole task

b: Component skill: constructing perpendicular lines

c: Component skill: constructing square

d: Strategy: Tracing the free point.



e: Component skill: constructing a ray, line or locus.

f: Component skill: constructing perpendicular lines

g: Component skill: constructing a square.

h: Strategy: exploring the problem space and evaluation

Figure 1: Inscribing a square within a triangle: the whole task, component skills, progression, and strategies.

Data Collection

A variety of data were collected during the implementations of the course through online discussion forums, on-demand brief questionnaires, and problem sets. Two pre- and post- content tests were also administered. At the end of the online course, a 25-item questionnaire (see Appendix A) on GeoGebra use was given to collect data on participants' self-perceived experience with GeoGebra in the context of the course. Each item has a six-point Likert scale ranging from 1 (*Strongly Disagree*) to 6 (*Strongly Agree*). A free-response question was also included to allow for comments and suggestions. Since there were no previous validated instruments to use, we reviewed similar survey instruments on the use of technology, such as calculators, in teacher education and professional development programs (Brown et al., 2007; Burrill, 1992; Guerrero, Walker, & Dugdale, 2004; Kastberg & Leatham, 2005) and adapted some of the questions to address our primary concerns and our primary research questions regarding the integration of open-source DIMLE technology. Based on our analysis of existing research findings in the literature, we established four tentative factors to investigate inservice elementary school teachers' use of GeoGebra—attitudes, curriculum, content, and pedagogy. With cohorts one and two combined, forty-five (45) out of a total of 53 teachers completed the GeoGebra questionnaire. In this article, we report the results of the questionnaire as a way to understand inservice elementary school teachers' experience with GeoGebra. We further conducted exploratory correlation analysis to gain insight into teachers' perspectives on the integration of GeoGebra in mathematics teaching and learning.

Results

Summary of the 25-item Survey

Under the above-mentioned instructional context and GeoGebra integration, we report the results of the four-factor questionnaire (N = 45) in **Table** 1- 4, which demonstrate the frequency and percentage of the responses for each item. A reliability analysis of the 25 items yields a Cronbach's Alpha of .874 and .900 based on standardized items. In light of the clear trends in the percentage clusters, we first summarize the major responses of our participants and then take a closer look at the correlations among the items.

Attitudes toward GeoGebra use. What attitudes do inservice elementary school teachers have regarding the use of open-source GeoGebra? As shown in **Table** 1, in the beginning, more than half (60%) of the teachers did not quite like the use of GeoGebra in the online course, which could be due to their lack of prior exposure and perceived difficulties with GeoGebra or similar technologies. However, after intensive course work and GeoGebra-based problem explorations, the majority (89%) of the teachers came to accept GeoGebra as a meaningful instructional tool and almost all of the teachers (98%) indicated that they shared or would share GeoGebra with their colleagues and the local community. More than 84% indicated that they would continue to use GeoGebra. The majority (76%) did not think that GeoGebra made mathematics more difficult to learn or teach.

Table 1: Attitudes Toward GeoGebra Use

Question	Very Strongly Agree	Strongly Agree	Agree	Disagree	Strongly Disagree	Very Strongly Disagree
At the beginning, I did not like GeoGebra at	28.9% (13)	11.1% (5)	20.0% (9)	**31.1% (14)**	4.4% (2)	4.4% (2)
Right now, I am more open to explorations using GeoGebra.	20.0% (9)	24.4% (11)	**44.4% (20)**	8.9% (4)	2.2% (1)	0.0% (0)
I like it because it is free to everyone.	**40.0% (18)**	22.2% (10)	37.8% (17)	0.0% (0)	0.0% (0)	0.0% (0)
I shared or will share GeoGebra with students and parents.	24.4% (11)	28.9% (13)	**31.1% (14)**	15.6% (7)	0.0% (0)	0.0% (0)
I shared or will share GeoGebra with my colleagues and administrators.	28.9% (13)	31.1% (14)	**37.8% (17)**	2.2% (1)	0.0% (0)	0.0% (0)
I will continue to learn and use GeoGebra.	26.7% (12)	24.4% (11)	**33.3% (15)**	15.6% (7)	0.0% (0)	0.0% (0)
GeoGebra makes mathematics unnecessarily difficult for me to learn and teach.	2.3% (1)	4.5% (2)	18.2% (8)	**34.1% (15)**	29.5% (13)	11.4% (5)

Curricular issues about GeoGebra use. What are the curricular challenges if teachers choose to use GeoGebra in their mathematics teaching? As shown in **Table** 2, more than half (67%) of the teachers felt that GeoGebra challenged their conception of mathematics. However, the majority (89%) felt that they could adapt existing materials for GeoGebra integration. The majority (87%) think that the teacher will have to face challenges from their own students when GeoGebra is used. Importantly, all of the teachers (100%) believed that GeoGebra helped students create their own mathematical ideas.

Table 2: Curricular Issues Related to GeoGebra Use

Question	Very Strongly Agree	Strongly Agree	Agree	Disagree	Strongly Disagree	Very Strongly Disagree
It challenges my conception of the mathematics I teach.	4.4% (2)	11.1% (5)	**51.1% (23)**	31.1% (14)	0.0% (0)	2.2% (1)
The textbook has to be revised for me to make better use of GeoGebra.	2.2% (1)	4.4% (2)	15.6% (7)	**53.3% (24)**	17.8% (8)	6.7% (3)
I can adapt existing learning materials for GeoGebra.	8.9% (4)	28.9% (13)	**51.1% (23)**	11.1% (5)	0.0% (0)	0.0% (0)
GeoGebra helps students create their own mathematical ideas.	8.9% (4)	28.9% (13)	**62.2% (28)**	0.0% (0)	0.0% (0)	0.0% (0)
The teacher will have to face the challenges from students when GeoGebra is used.	8.9% (4)	13.3% (6)	**64.4% (29)**	8.9% (4)	4.4% (2)	0.0% (0)

Mathematical content related to GeoGebra use. In terms of mathematical content, what are the major reactions of classroom teachers? As shown in **Table** 3, the majority (approx. 80%) of the teachers felt that they had re-learned some mathematical ideas or those which might be otherwise beyond their reach and that they were beginning to see mathematics as a consistent system. It is not surprising then that most teachers (78%) did not feel that GeoGebra made mathematics more difficult for them. However, the majority (80%) did feel a new type of mathematics was taking shape.

Table 3: Mathematical Content Related to GeoGebra Use

Question	Very Strongly Agree	Strongly Agree	Agree	Disagree	Strongly Disagree	Very Strongly Disagree
It helps me relearn some mathematical ideas.	6.7% (3)	24.4% (11)	**51.1% (23)**	17.8% (8)	0.0% (0)	0.0% (0)
It makes mathematics more difficult for me.	2.2% (1)	4.4% (2)	15.6% (7)	**57.8% (26)**	8.9% (4)	11.1% (5)
I have learned some mathematics that would otherwise be difficult to learn.	11.1% (5)	24.4% (11)	**44.4% (20)**	20.0% (9)	0.0% (0)	0.0% (0)
It helps me see mathematics as a consistent system of ideas.	11.1%(5)	20.0%(9)	**55.6% (25)**	11.1% (5)	2.2% (1)	0.0% (0)
I would like to learn more mathematics using GeoGebra.	13.3% (6)	17.8% (8)	**46.7%(21)**	22.2% (10)	0.0% (0)	0.0% (0)
I feel that a new kind of mathematics is being taught.	8.9% (4)	15.6%(7)	**55.6% (25)**	17.8% (8)	2.2% (1)	0.0% (0)

Pedagogical issues related to GeoGebra use. When GeoGebra is integrated, how do teachers perceive the pedagogical issues? As shown in **Table** 4, the majority of the teachers agree or strongly agree that GeoGebra helps them make connections among mathematical ideas (87%), reach out to more children (80%), rethink about mathematics teaching and learning (82%), create meaningful activities (89%), and provide instant feedback to students (96%). The majority of the teachers believed that their students generally like GeoGebra (89%) and indicated that they would be willing to learn with their students about GeoGebra (93%). One teacher wrote in the free response section, “My students really enjoyed seeing the things that we could create in GeoGebra. I have one student who has an aunt and an uncle who are math teachers and they contacted me about the program because she was so excited about it. I really think that this is a great way to get kids into math!” (March 20, 2010). Another teacher from Cohort Two wrote, “I am glad that I was introduced to the program and hope to continue learning and implementing the program in my classroom in the future!” (March 18, 2012).

Table 4: Pedagogical Issues Related to GeoGebra Use

Question	Very Strongly Agree	Strongly Agree	Agree	Disagree	Strongly Disagree	Very Strongly Disagree
It helps me make connections between different domains of mathematics	11.1% (5)	17.8% (8)	**57.8% (26)**	11.1% (5)	2.2% (1)	0.0% (0)
It helps me reach out to more children.	6.7% (3)	24.4% (11)	**48.9% (22)**	15.6% (7)	4.4% (2)	0.0% (0)
It helps me rethink about mathematics teaching and learning	13.3% (6)	22.2% (10)	**53.3% (24)**	8.9% (4)	2.2% (1)	0.0% (0)
It allows me to design meaningful activities for students.	13.3% (6)	17.8% (8)	**51.1% (23)**	13.3% (6)	4.4% (2)	0.0% (0)
Students generally like GeoGebra.	13.3% (6)	18.2% (8)	**59.1% (26)**	9.1% (4)	0.0% (0)	0.0% (0)
I am willing to learn with my students about the new tools.	20.0% (9)	17.8% (8)	**55.6% (25)**	4.4% (2)	2.2% (1)	0.0% (0)
GeoGebra constructions provide useful feedback to the students.	11.4% (5)	22.7% (10)	**63.6% (28)**	2.3% (1)	0.0% (0)	0.0% (0)

Correlation Analysis

To better understand the relationships among the 25 items on the GeoGebra survey, we obtained an inter-item matrix using SPSS®, which reveals two groups of items which are highly correlated ($R > .8$). The first group consists of Item A4, A5, and A6 (see appendix A):

A4: I shared or will share GeoGebra with students and parents.
A5: I shared or will share GeoGebra with my colleagues and administrators.
A6: I will continue to learn and use GeoGebra.

These three items are concerned with teachers' attitudes toward GeoGebra and its use in a learning community. It is not surprising that they are highly correlated. The second group of highly correlated items consists of Item D1, D2, D3, D4, and D7 (see appendix A):

D1: It helps me make connections between different domains of mathematics.
D2: It helps me reach out to more children.
D3: It helps me rethink about mathematics teaching and learning.
D4: It allows me to design meaningful activities for students.
D7: GeoGebra constructions provide useful feedback to the students.

These five items belong to the dimension of mathematical pedagogy centering round the idea of student engagement. It is again natural for them to be highly correlated in the data analysis.

Exploratory Dimension Analysis

Although our purpose is to understand inservice elementary school teachers' perspectives on GeoGebra integration in professional development as opposed to instrument design, we are curious about the major factors underlying our teachers' responses to the GeoGebra survey without adhering strictly to our literature review. With acknowledgement of the limitations of a small sample ($N = 45$), we explored the underlying structure of the 25 items using the method of exploratory factor analysis after removing highly correlated items A5, A6, D2, D3, D4, and D7 ($R > .8$).

Table 5: Rotated Component Matrix

	Component			
	1	2	3	4
A1	-.721			.501
A2	.573	.606		
A3		.677		
A4		.631		
A7		-.698		
B1				.760
B2		-.720		
B3		.626		
B4	.527		.581	
B5			.792	
C1	.596			
C2	-.512	-.703		
C3				.727
C4	.687			
C5	.690	.504		
C6	.682			
D1	.675			
D5		.535	.635	
D6	.578		.573	

Note. Extraction method: Principal Component Analysis;
Rotation method: Varimax with Kaiser Normalization.

After Varimax rotation with Kaiser normalization, we extracted four factors using Principal Component Analysis (**Table** 5). The factors account for 23.79%, 22.43%, 14.78%, and 8.40% of the variance, respectively, with a cumulative sum of 69.39% for the observed variance.

For an alternative perspective on our teacher participants' experience with GeoGebra, we could further consider the central theme of each factor (see **Table** 6). In spite of the overlapping of items, Factor One seems to cover the ideas about "the nature of mathematics," as is experienced by our teachers during their GeoGebra-integrated problem solving. If a teacher sees mathematics as a consistent and connected system of ideas, she or he would naturally be more open to explorative actions in the classroom and encourage students to proceed in the same direction. Factor Two seems to point to the theme of "instructional resource for mathematics teaching," which can be seen in the open accessibility of GeoGebra, its interactivity and dynamism, and teachers' perceived confidence in adapting existing curricular materials for use with GeoGebra. Factor Three seems to cover the theme that GeoGebra use changes the nature of "teacher-student interactions," which belongs to the domain of mathematical pedagogy. Finally, Factor Four seems to center round the growth of "teachers' knowledge of mathematics" in that it challenges their existing conceptions and helps them experience new ideas of mathematics.

In summary, the GeoGebra survey revealed at least four factors that are relevant to the use of GeoGebra in professional development: the nature of mathematics, instructional resource, teacher-student interaction, and teachers' knowledge of mathematics. Although the themes can be labeled in a different manner, we believe that survey results and findings of the subsequent exploratory factor analysis are consistent with the central themes in the literature. As the GeoGebra community strives to enhance the quality of mathematics education at an international scale, these central factors are worthy of consideration in program planning, implementation, and evaluation.

Table 6: Four Factors and the Related Question Items

Factor	Question Items
One	A1: At the beginning, I did not like GeoGebra at all. A2: Right now, I am more open to explorations using GeoGebra. B4: GeoGebra helps students create their own mathematical ideas. C1: It helps me relearn some mathematical ideas. C2: It makes mathematics more difficult for me. C4: It helps me see mathematics as a consistent system of ideas. C5: I would like to learn more mathematics using GeoGebra. C6: I feel that a new kind of mathematics is being taught.
Two	A3: I like it because it is free to everyone. A2: Right now, I am more open to explorations using GeoGebra. A4: I shared or will share GeoGebra with students and parents. A7: GeoGebra makes mathematics unnecessarily difficult for me to learn and teach. B2: The textbook has to be revised for me to make better use of GeoGebra. B3: I can adapt existing learning materials for GeoGebra. C2: It makes mathematics more difficult for me. C5: I would like to learn more mathematics using GeoGebra.
Three	B4: GeoGebra helps students create their own mathematical ideas. B5: The teacher will have to face the challenges from students when GeoGebra is used.
Four	A1: At the beginning, I did not like GeoGebra at all. B1: It challenges my conception of the mathematics I teach. C3: I have learned some mathematics that would otherwise be difficult to learn.

Concluding Remarks

As shown in the questionnaire results and the above discussions, our inservice teachers benefited significantly from the use of GeoGebra in various ways, ranging from personal mathematical exploration, attitudes toward mathematics and mathematics teaching, to pedagogical reflections, including the nature of mathematics and teacher-student interactions. These changes are well aligned with the emphases of the ongoing mathematical education reform, including the integration of technology. Our preliminary findings support the use of GeoGebra or similar DIMLEs in professional development programs that seek to enhance inservice teachers' understanding of the big ideas of mathematics and further to empower them with the pedagogical tools to enact changes in their teaching practice. Looking forward, we would like to make the following recommendation for future endeavors:

(1) Provide cognitive support for teachers who may have initial, self-perceived, negative reactions to GeoGebra or other new innovations, which are a necessary step toward informed and sustainable use of GeoGebra to improve mathematics teaching and learning.

(2) Develop self-contained online video-based tutorials showcasing genuine mathematical problem solving, especially in online learning cases. These video modules can be watched by teachers on demand. As one of our teachers pointed out, "[t]he videos are a must for someone just learning how to use GeoGebra. I can watch them as I need to WHEN I need to. I have watched them over and over. Each time, I pick up a new piece of information."

(3) Encourage and support teachers in using GeoGebra with their own students. As our teachers observe children's mathematical exploration with technologies, they may learn about themselves as problem solvers and develop appropriate strategies to support student learning.

(4) Use genuine mathematical problems in GeoGebra-integrated learning. Not all problems are best investigated with GeoGebra. Problems should be carefully selected or designed according to the mathematical maturity of participants, to maintain a desirable level of cognitive complexity and pedagogical flexibility (Doerr & Pratt, 2008; Martinovic & Karadag, 2012).

In closing, we would like to point out the importance of instructional theories in our overall course design and daily interventions. Professional development is a highly complex endeavor, which gets more complicated when dynamic technologies are intertwined with mathematical and pedagogical issues. Well-designed GeoGebra integration may eventually help control the complexity and provide genuine mathematics learning experience to teachers and students alike.

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Appendix A: GeoGebra Survey and Item Coding

Code	Question Item
A1	At the beginning, I did not like GeoGebra at all.
A2	Right now, I am more open to explorations using GeoGebra.
A3	I like it because it is free to everyone.
A4	I shared or will share GeoGebra with students and parents.
A5	I shared or will share GeoGebra with my colleagues and administrators.
A6	I will continue to learn and use GeoGebra.
A7	GeoGebra makes mathematics unnecessarily difficult for me to learn and teach.
B1	It challenges my conception of the mathematics I teach.
B2	The textbook has to be revised for me to make better use of GeoGebra.
B3	I can adapt existing learning materials for GeoGebra.
B4	GeoGebra helps students create their own mathematical ideas.
B5	The teacher will have to face the challenges from students when GeoGebra is used.
C1	It helps me relearn some mathematical ideas.
C2	It makes mathematics more difficult for me.
C3	I have learned some mathematics that would otherwise be difficult to learn.
C4	It helps me see mathematics as a consistent system of ideas.
C5	I would like to learn more mathematics using GeoGebra.
C6	I feel that a new kind of mathematics is being taught.
D1	It helps me make connections between different domains of mathematics.
D2	It helps me reach out to more children.
D3	It helps me rethink about mathematics teaching and learning.
D4	It allows me to design meaningful activities for students.
D5	Students generally like GeoGebra.
D6	I am willing to learn with my students about the new tools.
D7	GeoGebra constructions provide useful feedback to the students.