

## Introducing parameters of linear function at undergraduate level: use of geogebra

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<b>Article history</b>	<p>The newly founded universities increase the number of high school graduates taking a seat in a university class. Among them the ones who are not well prepared in mathematics, mostly in the social sciences students, face learning difficulties in the first year mathematics course at undergraduate level. The notion of the function which is the foundation of the first year undergraduate mathematics course is a difficult part of mathematics curriculum especially for the under risk new students of social sciences. To overcome the problem, an intervention is applied in the course as Geogebra creating visual learning environment. Teaching of function is organized with the method of contextual teaching and learning (CTL) by using the REACT strategy. The instruction is started with the Relating, the first step of strategy, to reach each student's background to relate it with their daily life. Critical Incident Questionnaire (CIQ) is used to get students' reflection about the lecture and has used the reflections, especially the ones who see the point of relating, to form the sample group of the study. The activity including geogebra is developed to apply second step of strategy as Experiencing. The task is organized as to investigate the effects of change in the parameters of a linear function. After analysing research documents, findings reveal that students have benefits from two important words as visual and concrete in dealing with Geogebra as a member of Dynamic and Interactive Mathematics Learning Environments (DIMLE).</p>
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### Introduction

The Higher Education Area in the world is expanding for the need and advantage of reaching the job opportunities. The same trend is occurred in Turkey as opening of new universities with a strategy, one for each city as a government policies (YOK, 2007; Kavak, 2011). The new universities give a chance to students who do not enough mathematical background because of the structure of the university entrance exam. The university exam puts the students in order by taking their exam results into account till to place every student in university regarding only the quota without checking the prerequisite of the departments. Thus, the students even who have never solved any mathematics questions may have the place in a department, demanding mathematics background.

In this context, universities should decide how to manage the problem of unprepared students or wait the high schools to give more ability students to universities. In the world, universities

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do their solutions as opening remedial classes, support centres, etc. In this study, as the researcher and the lecturer of mathematics course, I focus on course teaching not outside solution as mentioned above. Rather, I use action research method to make the problem clear and its solution from inside as applying interventions during the research cycle.

### **REACT Strategy**

To overcome the problem, an intervention is applied in the course as Geogebra creating visual learning environment. “Why something is done in a particular way?” is a strategic question that puts the distinction between high school where procedural learning plays a major role and university where relational understanding (Jaworski, 2013), therefore, teaching of function is organized with the method of CTL by using REACT strategy (Crawford, 2001) developed by Centre of Occupational Research and Development (CORD). The use of CTL, combines teaching and learning with learner’s own life situations (Baki & Cathioglu, 2008) in lecture with the strategy has the following steps in order; Relating, Experiencing, Application, Cooperating and Transferring. In the experiencing step of the strategy, Crawford expresses the importance of students’ background as daily life experience and its relation to mathematics should grow and interact with hands on activities. The lecture should start with what students already know instead of what teacher knows (Baki, 2009) and students’ background knowledge should be not enough for the course. Çathioğlu (2010) states that learners’ daily life backgrounds would not be the same, that’s why experiencing step is reorganized by the researcher from the perspective of the Realistic Mathematics Study. Thus, the daily life experience is excluded and an experimentally experienced model is included as suggested by Barnes (2005). The reason for this change is to encourage students to explore mathematics.

### ***Dynamic and Interactive Mathematics Learning Environments: Geogebra***

Reconsideration of the second step of REACT strategy, wide use of Geogebra software is found relevant for experimentally designed lectures. The visualization features of Geogebra provides opportunities for students to make mathematical concepts visible (Little, 2001), to experiment mathematics (Lavicza, 2010) in dynamic and interactive mathematics learning environments (DIMLE), and to improve their mental schemas for the concepts (Martinovic & Karadag, 2012). The realistic and experiencing steps are linked by employing Geogebra to bridge learners’ existing knowledge to new concepts (Berry & Nyman, 2003) and knowledge in the brain appears as concrete and interactive to test learners’ estimation about the task in computer environment (Akkoc, 2006). Hence, Geogebra increases the possibility to experience learning objects (Attorps, Bjork & Radic, 2011). The link between experiencing and application steps, constructed by using computer environment, increase undergraduate students’ performance on calculus concepts (Naidoo & Naidoo, 2007) and computer aided instruction increases the achievement of learning outcomes as well as learners’ motivation (Yilmaz & Guven, 2011).

### ***The Study***

In the study, experiencing step is applied by using Geogebra from the perspective of Jaworski (2013) to make the abstract concepts more visible to enhance participants’ learning. The general aim of the study is to overcome the problem of under risk students’ mathematics performance by the CTL method via providing DIMLE: Geogebra is used to support the main aim of the study and to accomplish the experiencing step of the REACT strategy.

## Method

This study is a 2-step qualitative research and designed to illustrate the problem and a possible solution for the problem from as the insider researcher and the lecturer of the course. To improve my teaching experiences in education settings, Pedagogic Action Research for universities, including steps of identifying problem, thinking about the solution, applying solution, evaluating results and modifying procedure, (Norton, 2009) is used. Norton (2009) explains the reliability of action research from generalizability perspective as reliable for the similar situations and validity as discussing the results with the researchers. As stated above, situation of unprepared students in mathematics classes may appear in new universities and the paper itself opens the feedback channel to other researcher to discuss the research findings.

## Study Group

In order to form study group, after first step of the React strategy; Relating, the CIQ is distributed to 220 first year students of economics and business administration departments. The study group consists of 30 volunteer students, chosen from each department in Faculty of Economics and Administrative Sciences in Bayburt University in the academic year of 2012-2013 fall semester.

## Data Collection Tools

In the study, data collection tools are the focus group interviews, CIQ, problems addressing the informal definition of function, slope and Geogebra task for changes on the graphs. The plan of the study is given Table 1.

**Table 1.** Plan of the study

REACT	Data Collection	Type
Relating	Problem case of the informal definition of function	Class Activity
	The Critical Incident Questionnaire	Tool
	Problem case of the slope	Class Activity
Experiencing	Geogebra task of change	Task
	Focus Group Interview	Tool

The notion of function which is the fundamental for the first year undergraduate mathematics course is a difficult part of mathematics curriculum (Basturk, 2006), particularly for the unprepared new students of social sciences. Informal definition of function is given for the problem case (Figure 1.) to relate existing knowledge to new knowledge that is the Relating step of the REACT strategy.

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The amount of consumption, unit price and amount to be paid is written in the bill.

1. Find the variables and parameters.
  2. Decide the dependent and independent variables.
  3. State the relation between dependent and independent variables
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**Figure 1.** Problem case of the informal function definition

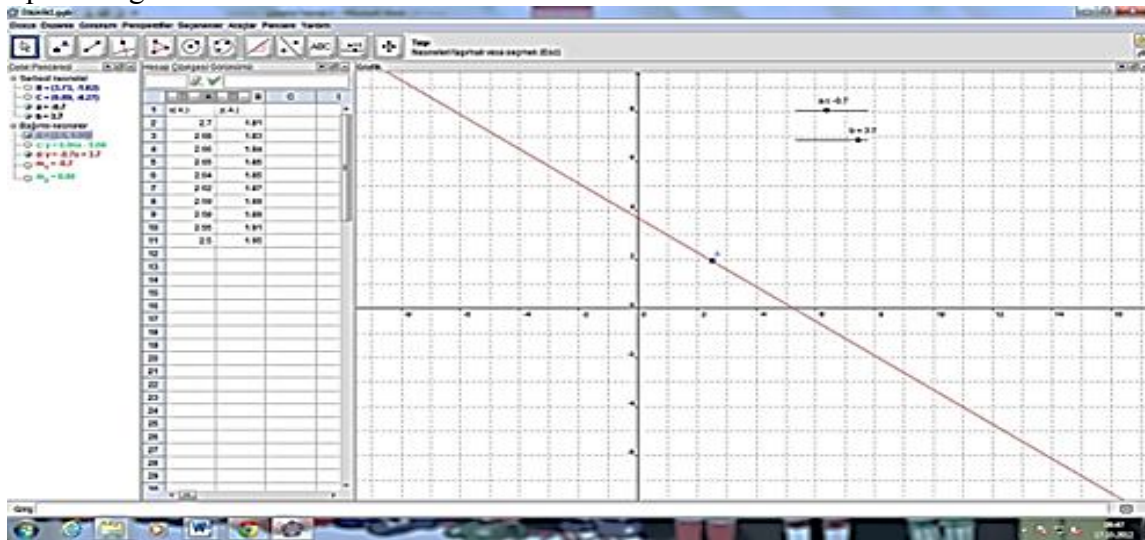
At the end of the lecture, the CIQ is applied to whole class. Brookfield (1995) uses the CIQ as a post-class formative assessment tools. This side of CIQ is also suitable for action research for interventions during the research process that differs from experimental design research. CIQ has five basic questions and applied after the course delivered by the lecturer. Incidents happen but critical incidents are produced (Angelides, 2001) so the analysis of CIQ contains meanings given to the reflection of students.

After giving the informal definition of the function, the concept of slope is given with the problem case (Figure 2.). The aim of this task is to relate slope and three different changes (increasing, constant, decreasing) on the graph of the function.

1. Graph the height of a plane flying from Trabzon to İstanbul wrt time.
2. Teacher throws the chalk to up and then holds it. Graph the height of the chalk wrt time.

**Figure 2.** Problem case of the slope

The relating step of the strategy is to accomplish with two tasks, one for daily life while the other experimental situation. So, now it is time to move forward to the second step as experiencing. The slope activity is developed in Geogebra for students to experiment the change in parameters. The Geogebra task named “Change” (Figure 3.), including questions to lead conclusions (Kutluca & Zengin, 2011), is uploaded to [www.geogebra.org](http://www.geogebra.org) website by user name PhD and the class worked on it in the faculty’s computer laboratory under the supervising of the researcher.



1. Giving different values to the parameters “a” and “b” by sliders a and b, investigate the changes on the graph of the function.
  - 1.1 State the relation between the parameter “a” and the graph by keeping the parameter constant.
  - 1.2 State the relation between the parameter “a” and graph keeping the parameter constant.

**Figure 3.** Geogebra task of  $y = a x + b$

The aim of the task is to help social sciences students develop on understanding of the parameters and their changes on graph and to open windows to the meaning of limit concept as approaching and derivative concept as the bridge between average and instant changes.

## Results

### *The Critical Incident Questionnaire*

Some of the CIQ are as follows: questions and the reflections for those questions;

1. At what moment in class did you feel most engaged with what was happening?  
*... when we all do comments, ... continuously being questioned by lecturer, ... when we discuss, ... examples from daily life, ... to be in dialogue with classmates, ... to understand what the subject is related to*

2. At what moment in class did you feel most distanced from what was happening?  
*... when lecturer states function verbal, ... besides high school, here I understand what to do with what I learn*
3. What action that anyone (teacher or student) took in class did you find the most affirming or helpful?  
*... teaching with daily life, ... concrete examples, ...written and verbal teaching, ... listening the answers from students, ... to be in dialogue with everyone, ... comprehend subject by imagination*
4. What action that anyone (teacher or student) took in class did you find the most puzzling or confusing?  
 NA
5. What about the class surprised you the most?  
*... asking for verbal definition, ... sadden to afraid of mathematics during twelve years, ... daily life examples, ... I heard everything for the first time, ... the things that I could not do till now, I taught I can*

Based on the incidents that students provide from reflection of the course, the researcher produced critical incidents (Table 2.).

**Table 2.** Critical Incidents produced by the author

Critical Incidents	
1	Daily life (concrete) examples of informal teaching
2	Discussing the topic as a class with questioning continuously
3	Motivation

### Geogebra Task

The investigation of the Geogebra task for changes on the graph, students have used the following words; increasing, decreasing, position, changes, move, angle between, approaches, towards, slides, turning and diagonal. Such a range shows how students interlink their existing knowledge to new one in their minds as individual mental schemas (Martinovic & Karadağ, 2012).

The first task of Geogebra “State the relation between the parameter “a” and graph keeping the parameter fixed.” yields the answers according to the different values of the parameter “a”, some are listed in Table 3. The words students prefer to use such as increasing, changes, moves, towards, turning and slides shows the way how students recognize the change in their minds and states it in words from daily life repertoire.

**Table 3.** Answers for parameter “a”

Parameter	Changes on Graph
$a > 0$	Increasing to left, On 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant, Changes positive oriented, Increasing upward with slope, When value of parameter increases, it moves upward in positive way, On 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant tends to right, Moves in counter clockwise direction, Angle between y-axis is decreasing, Moves upward diagonally, Approaches to y-axis on 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant, Towards left vertically, Slides to left, Turning to left
$a = 0$	Constant, Continuous in horizontal axis, Parallel to x-axis, Steadily up and down, Constant on horizontal axis, On 1 <sup>st</sup> and 2 <sup>nd</sup> quadrant parallel to x-axis, Linearly, Horizontally, On plane linearly fixed
$a < 0$	Increasing to right, On 2 <sup>nd</sup> and 4 <sup>th</sup> quadrant, Changes positive oriented, Increasing downward with slope, When value of parameter decreases, it moves in negative way,

On 1st quadrant, Approaches to y-axis on 2nd and 4th quadrant, Angle between y-axis is decreasing, On 2nd and 4th quadrant tends to left, Moves in counter clockwise direction, Moves downward diagonally, Towards right vertically, Slides to right, Turning to left

The second task of Geogebra “State the relation between the parameter “b” and graph keeping the parameter constant.” yields the answers according to the different values of the parameter “b”, some are listed in Table 4. Beside the words students used, the new word such as pass through, at the centre and cuts are seen in Table 4.

**Table 4.** Answer for parameter “b” results

Parameter	Changes on Graph
$b > 0$	Increasing to left, On 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant, Moves positive direction, Moves upward increasingly, On the 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant tends to right, On 2nd quadrant, Moves to left, Moves to left paralleled, Increasing to left upward, Slides to right on 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant, Slides to left, Moves to infinity, Moves to 2 <sup>nd</sup> quadrant, Slides upward, Moves away from origin as x is increasing, Moves negative side of x-axis and positive side of y-axis, Most part of it is on 2 <sup>nd</sup> quadrant
$b = 0$	Pass through origin, Moves horizontal axis, On the 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant tends to right, On 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant tends to right, At the centre of zero, $y = x$ , Diagonal in opposite sides of x-axis, Crosses origin at right angle, Cuts coordinate axis symmetrically, Moves diagonally, Constant at the origin
$b < 0$	Moves downward, On 1st and 3rd quadrant, Moves negative direction, Moves upward decreasingly, On 4 <sup>th</sup> quadrant, On the 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant tends to right, Moves to right, Becomes parallel on the left, Slides to right, Moves to infinity, Moves to 4 <sup>th</sup> quadrant, Slides downward, Moves away from origin as x is decreasing, Moves positive side of x-axis and negative side of y-axis, Least part of it is on 4 <sup>th</sup> quadrant, Slides to right on 1 <sup>st</sup> and 3 <sup>rd</sup> quadrant

**Focus Group Interview Data**

The investigation of the focus group interview data reveals the voice of students about the benefits of Geogebra to be in four categories.

Category 1. Exam Dominated: Students take Geogebra from exam perspectives as “I understand in lab, but I can’t do in exam” and “In exam, lab activity came to my mind, not the one on the board in class.”

Category 2. Logic Behind Topic: Students remarks Geogebra as a tool to help understanding the logic behind the topic as “Lab supports one-on-one what I learn in class.”, “I understand what I do.”, “When it is repeated in class, everything matches.”, “We do everything that I can’t forget.” and “It is better than memorization the rules.”

Category 3. Visualization: Students remarks the concrete side of Geogebra as “I see everything and I manage it myself.”, “I see everything in details.” and “I see what is changing when I change.”

Category 4. No Benefits: Some students do not see any benefits of Geogebra because of reasons not related to Geogebra as they compare lab and class by “You teach very well in class.”, they are not to be familiar with computer and software and the last reason is they prefer memorizing rather than understanding.

## Conclusion and Suggestions

In this study, my aim is to use Geogebra in reconsidered version of experiencing step of the REACT strategy. The CIQ results show what students appreciate in teaching that one is about the experimentally concrete activities, Geogebra task responds demonstrate a huge range of words including concrete thoughts about the change on graph and the focus group interviews show the logic and the visualization of the concepts. When we put all these three results in one word, concrete is the magic word can be seen during the research period. This part of research results shows the innovation's difference from traditionally teaching where no connections to non-mathematics major students' professional interests (Abramovich & Grinshpan, 2008), where the study group is composed of social sciences. Concrete makes student to understand the topic, grasp the logic behind the course and let them to understand what is taught in the course. Geogebra plays a bridge role between Relating and Experiencing steps to be united to carry out students for the next step of the REACT strategy as similar to what Jaworski (2013) stated.

The first cycle of action research is applied with the reconsidered version of experiencing step. The results stated above make new plan for following cycle of research as considering when to apply Geogebra task, before or after class activity. Since some students in interview mention about advantage of doing Geogebra task before class activity whereas the others prefer before the class activity.

For lecturer to make students understand the concepts, Geogebra gives them a chance to develop tasks that combine visual and concrete side of learning that fosters the teaching for low attainers in mathematics that matches interactive and contextual approaches at tertiary level (Biggs, 2006).

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