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The effect of Dynamic and Interactive Mathematics Learning Environments (DIMLE), supporting multiple representations, on perceptions of elementary mathematics pre-service teachers in problem solving process

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Mathematics is an important discipline, providing crucial tools, such as problem solving, to improve our cognitive abilities. In order to solve a problem, it is better to envision and represent through multiple means. Multiple representations can help a person to redefine a problem with his/her own words in that envisioning process. Dynamic and Interactive Mathematics Learning Environments (DIMLE), allow multiple representations in mathematics and therefore provide opportunities to explore, to explain, and to model (Karadag & McDougall, 2009) mathematical subjects or concepts. The goal of this study is to explore the effect of multiple representations on perceptions of elementary mathematics pre-service teachers in problem solving process. Hence, ten representations, which dynamically represent a word problem, are created by using one of the DIMLE, GeoGebra. The sample group consists of elementary mathematics pre-service teachers (n=17). In order to precisely analyze the data, which are gathered from sample group, quan-qual mix method was used. Consequently, the use of multiple representations, which were prepared by DIMLE, cause to a mental demand for the other mathematical concepts except for understanding and solving a problem.

Introduction

Mathematics is a prominent discipline with its abstract concept, different thinking skills and compatibility with real-world. The tools of mathematics are not only used for its concepts but also used for other disciplines and social life. One of these tools, may be the most important one, is problem solving.

Problem solving has a central importance for mathematics and it is accepted as a fundamental tool for understanding and interpreting mathematical knowledge (Jitendra, Griffin, Buchman & Sczesniak, 2007; Kayan ve Çakıroğlu, 2008; MEB, 2005; NCTM 2000, 2004; Polya, 1957; Schoenfeld, 1987). This fundamental tool is also a special process rather than a concept or subject. That process involves, verbal and syntactic processing, visualizing; building different type of representations, algorithmic processing, algorithmic learning, and debugging; the use

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of mathematical notations; conceptual understanding; the detection of structures and structural similarities; change of representation; transfer in between different types of representations, metacognitive process (Goldin, 1992). In other words, problem solving is not just calculating numbers or completing numerical operations. People use their mathematical thinking skills and cognitive tools in this solving process (Mason, Stacey & Burton, 1985). Even though people have adequate mathematical knowledge to solve a problem, they need a strategy in order to clarify their actions for problem solving. Polya (1957) described a strategy, which contains four steps, for problem solving process. These are "understanding, planning, implementing and looking back". "Understanding" is the most important action in problem process (Cai, 2003; Garderen & Montague, 2003; Jitendra, Griffin, Buchman, & Sczesniak, 2007; Karataş & Güven , 2004; Mayer, 1982; Polya, 1957; Stoyanova, 2005). In order to solve a problem it is necessary for people to have a good understanding of it. Polya (1957) emphasizes that if the solver has not a firm understanding of what the problem is about, the solution cannot possibly occur. When solver can comprehendingly combine his/her knowledge with a solving strategy, the transformation and/or exploration actions, which can be applied to the problem situation, can emerge. Representations play a significant role in the emerging of that perspective (Cai, 2005; Cifarelli, 1998; Goldin, 2002). Representations are the tools, which illustrate mathematical concepts verbally, numerically, and algebraically (Schneider, 1995). The visual presentation of knowledge is accepted as an effective way in teaching mathematics and especially in problem solving process (Presmeg, 1986; Zimmerman & Cunningham, 1991). Therefore, representations have a crucial role in enhancing mathematical thinking (Confey & Smith, 1991), and developing problem solving skills (Lubinski & Otto, 2002). Mathematical thinking, from elementary to advanced level, involves a significant transition such as describing to defining, from convincing to proving in a logical manner based on those definitions (Tall, 1991). Because of adding a new dimension of understanding into the problem solving process, representations may be accepted as an important tool for that transition in problem solving process.

Multiple Representations

Multiple representations are generally defined as representing mathematical relationship in different ways (Özgün-Koca, 2004), such as symbol, diagram, table, verbal statement and figures etc. Similarly, NCTM (1989) defines multiple representations as the key elements in mathematics education. Keller and Hirsch (1998) emphases that the use of multiple representation is an advantage, because multiple representations avoid limitations of one type representation and building the new one, which is more perceptible and more helpful for problem solving process. Hiebert and Carpenter (1992) and Kaput (1998) indicate that the use of multiple representations helps students learn by employing their own thinking and learning habits. Because of differences in students' learning style, they need to use more than one type of representation so that they can create their own solving environment. Multiple representations provide visualization for those solving environments, thus students can make their solving actions visible. Those representations also help for re-questioning those actions and also their thinking style.

Dynamic and Interactive Mathematics Learning Environments (DIMLE)

After the development in Information and Communication Technologies, a new generation rose, so-called Net Generations (Oblinger, 2005; Tapscott, 1998), Digital Natives (Prensky, 2001) and etc. They are identified as the visual leaners and also described as the people learning differently than their predecessors. Because of those differences they need new learning environments, which are compatible with their learning skills and contains an immediate interaction.

Dynamic Interactive Learning Environments (DIMLE) (Martinovic & Karadag, 2012) is the name for specifically designed digital platforms providing learners special opportunities to increase mathematics knowledge and understanding levels. Cabri, GeoGebra, Geometer Sketchpad, Fathom and the like are some of the DIMLE (Martinovic & Karadag, 2012). DIMLE provide those learners an explorative world and more than just one concept in examples (Martinovic, Freiman & Karadag, 2013). Thus DIMLE give them a chance to build their own mathematical constructions depending their exploration and curiosity.

DIMLE are developed for supporting the learning mathematics through free exploration in the less constrained environments (Martinovic & Karadag, 2012). However carrying out that exploration and taking the advantages depend on the realizing key qualifications of DIMLE.

Typical DIMLE software contains interactivity and dynamism as the key features. Interactivity means a fast feedback mechanism, which contains action and reaction iteration. Dynamism is related to continual change in the process. Both features are important to support learning process and to teach fundamental dynamic mathematical concepts by using technology (Martinovic & Karadag, 2012). DIMLE provide a contact between multiple representations and mathematical abstract concepts. Thus it causes to develop the students' visualizing skills and to increase conceptual understanding (Ayvaz Reis & Özdemir, 2011; Velichova, 2011; Yuyucu & Ayvaz Reis, 2010).

The Study

The goal of this study is to identify the effect of multiple representations on perceptions of elementary mathematics pre-service teachers in problem solving process.

Dominant - less dominant quantitative - qualitative sequential mixed method was used in the study. In the less dominant quantitative part, descriptive model was employed to document the sample group's preferences about representations. In the dominant qualitative part, case study model was used to observe sample group's exploration process, their actions and reactions about representation in problem solving process.

The sample group consists of elementary mathematics pre-service teachers (n=17) in Hasan Ali Yucel Educational Faculty at Istanbul University. The homogeneity criteria of that group are;

- To have the same elementary mathematics knowledge level
- To successfully graduate fundamental computer lessons
- To successfully take the courses of "school experience 1" and "school experience 2"
- Not to have any information about screen capturing technique and DIMLE.

Data Collection Tools

The notes and worksheets of sample group in evaluating process of representations, open ended questions about their perceptions and understanding process, voice and video records of sample group and screen captures of their computer screens are the data collected for this study.

Conducting the Research

After specifying the sample group, the training was provided for the participants. Meanwhile in order to determine the effect of DIMLE, a word problem that can be solved by using first order equations with one-two unknowns, was chosen. 10 different representations, which described that problem, were designed. The design of those representations was based on a dynamic structure. That dynamic structure was constructed by using a GeoGebra tool, sliders, and building the logical relationship between sliders and the elements of problem. Table 1 illustrates the presentation types used in the study.

Table 1. The presentation types of problem in representations.

Representations	Representation Types of Problem
Representation 1(R1)	figures
Representation 2 (R2)	x
Representation 3 (R3)	Colored lines
Representation 4 (R4)	Black lines
Representation 5 (R5)	words
Representation 6 (R6)	Given - asked
Representation 7 (R7)	Question marks (?)
Representation 8 (R8)	Black-white boxes
Representation 9 (R9)	Black boxes
Representation 10 (R10)	Colored boxes

In order to validate the representations, we interviewed with 29 mathematics teachers. According to their teaching experience, those teachers can be separated into three type of group. One of them is professional, who has experience over 10 years. The other one is an experienced teachers, who have experience between 5 and 10 years. The last one is a novice teachers, who are just graduated or have experience less than 1 year.

We asked those teachers that "how many way can be used, when we present a word problem" and "what kind of representations can be designed". According to their answers, those representations were reviewed and some of them reorganized.

In order to ensure the reliability of training process, a pilot application was fulfilled with one person, who had the same qualification with the sample group. According to data, which were procured from pilot application, the training process and representations were controlled. After the pilot application process was completed, the main application was fulfilled. In that application process, the sample group was trained about GeoGebra and screen capturing technique for a four-week period. At the end of the training, the chosen word problem and representations were given to sample group. Therefore, they examined that problem by using pre-designed representations. In that examining process, the behaviors, reactions and speaking of sample group were recorded.

Findings and Discussions

The relationship between representations and word problem was asked to the sample group, and then, their reactions were observed and analyzed. Table 2 illustrates the reactions and actions of sample group.

Table 2. The reactions and actions of sample group

Evaluations of Problem and representations	Evaluation Types
Evaluating the problem by depending on representations	Researching relationship between the numbers
	Questioning the sliders
	Indecision, depends on result of problem

Evaluating the problem by being independent on representations	Digressing the problem

We observed that all teachers, in the sample group, firstly wrote the answer of that problem without stating an equation. After replying, they checked the representations in order to validate their answers. Because of the dynamic structure of the problem, in representations, they could not find their answers firstly, and then they started to complain about representations and reacted.

The first reaction of the sample group was to analyze the representations by accepting the results of problem as constant and unchangeable. Thus the following reactions, in sample group, were observed:

- Complaining about representations (because of dynamic structure)
- Accepting the representations were wrongly designed
- Examining the inconstancy in the representations.

After the examining numbers in representations, they started a similar examination in sliders. Thus those reactions, in sample group, were observed:

- finding out the relationships between sliders and the result of the word problem
- trying to describe a mistake not numerical but total in that problem

We tried to understand their analyzing process in changing numbers in sliders. Table 3 illustrates the analyzing style of the sample group in that process

Table 3. The analysing style of the sample group

Analyzing Style	Analyzing Action	Number of people
Symbolic	Writing equations, depends on the movement of slider	16
Modal	Drawing representations on papers	4
	Drawing sets	1
	Painting square shapes	1
Graphical	Marking dots on coordinate system	8
	Drawing lines on coordinate system	8

Table 3 shows that 16 people, in sample group, used mathematical symbols in order to explain the complexity in their mind. One in sample group was kept out, because she could not construct an equation form.

We asked for the sample group, that “what kind of mathematical thoughts were developed in their minds except the word problem, when they were analyzing the representations”. Table 4 illustrates mathematical thoughts of sample group

Table 4. Mathematical thoughts of sample group

Main Idea	Sub-idea
Problems and equations	Different type of one degree problems
	Equations
Coordinate system and its elements	Ordered pairs
	coordinate axis
Lines	Translation
	Type of lines
	Notations of lines
Functions	Definition of functions
	$x \rightarrow f(x)$

Ratio-proportion	Direct proportion
Slope	Lines
	Trigonometry
Similarity	Similarity in triangles

The sample group tried to find out new mathematical subjects and then they explored new mathematical thoughts. In that exploration process, they demonstrated the following reactions:

- Writing complete equations, which observed in movement of sliders (17 people)
- Analysing unknowns (17 people)
- Organizing unknowns by using ordered pairs (14 people)
- Marking those pairs in coordinate system (8 people)

Additionally the sample group started to explore:

- the relationships between equations and coefficients (5 people)
- slopes in the lines (1 people)
- intersections between the lines (1 people)
- similarities in geometric shapes, which were came from intersecting lines (1 people)

We ask the sample group to put the representations in order from the most interesting one (10 point) to the less interesting one (1 point). Therefore, this process provided us some quantitative data. The scores of sample group were analyzed; Table 5 illustrates the average scores of whole representations.

Table 5. The average points of whole representations.

Representations	Average Points
R1	9.47
R2	4.47
R3	6.70
R4	5.41
R5	6.47
R6	5.00
R7	5.47
R8	4.35
R9	4.35
R10	6.70

It was found that the R1had the highest point (14 people). Hereby it was chosen as the best representation for that problem. One of the most important factors, which affected the sample group's approach, were representation type of the problem, such as colorfulness, using figures and/or symbols. When the choices of sample group were analyzed, it was brought out that they preferred colorful representations more than white-black ones. Scaife and Rogers (1996); indicates that colorful representations were more affected than white-black representations

Conclusion

In this study the effect of the multiple representations, provided by DIMLE, on perceptions of elementary mathematics pre-service teachers in problem solving process was explored. In the study, the different representations were created for the same problem. Thus the trigger factors could be clarified. However Meltzer (2005) revealed that different representations for the same problem caused the difference in the performance of students.

Different representations may give them a chance to build up different approaches to the same problem. Thus students did not stick in one idea and also to one representation. To construct a verbal and visual description of same problem can give chance to convert this into a mathematical representation.

When the problem evaluating processes of the sample group were analyzed, it was realized that that main process could be separated to stages. Those stages, we called, were habits-expectations stage, hesitation stage, chaos-denial stage, chaos-curiosity stage and exploration stage.

In habits-expectations stage, the sample group just tried to find the answer. They also symbolized the problem by ignoring representations, just directly answered "the asked value", and searched that value in representations. In hesitation stage, the sample group had incompatibility between what is given and what is seen. Thus they started to realize differences. They also tried to form representations according to the problem. In chaos-denial stage, they claimed that representations were wrong, and they also try to prove their opinions. In chaos-curiosity stage the sample group started to give the meaning the changes and differences. The exploration stage refers to last questioning level of the group. The sample group was separated to three phases. Those phases are phase 1-jamming, phase 2-modelling, phase 3- discovering phase. Phase 1 (jamming phase) refers to three actions, those are to have doubts about representations and own self, and to question. Phase 2 (modeling, phase) refers to two actions, those are to refocus and to model. The last phase, phase 3 (discovering phase) refers to two actions; those are to ignore limitations, to think the other side of representations and problem and to satisfy.

14 of sample group reached to discovering phase in the exploration stage. At the end of study the problem was not just a problem anymore for sample group. It is cleared that the multiple representations, which were prepared by using DIMLE, are more helpful than the classic multiple representations. Likewise the studies of Forster (2006), Goldman et al. (1999), Santos-Trigo (2004), Yerushalmy (1991) showed that using DIMLE multiple representations improved the problem solving abilities of the students.

Consequently, the use multiple representations, provided by DIMLE, diversified the sample group's approach to the problem. These results are along the same line with the studies, which contain the use of one of DIMLE, Sketchpad (Dugdale, 1999; Goldenberg & Cuoco, 1998).

DIMLE provide opportunities to visualize mathematical knowledge and to have immediate feedbacks. For example, multiple representations, some of these opportunities, provided by DIMLE, may help students be better problem solvers, to critically think, to create and to use different type of representations effectively. DIMLE may also help for thinking beyond the limitations of only the "given" concept and/or problem. It may cause a mental demand for the next concepts, thus one's mind may become ready to learn and already open to think dynamically. DIMLE may give opportunities to students in order to build their own mathematical thinking world, where they keep there should be more.

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