Mevlana International Journal of Education (MIJE)
Vol. 1(2), pp.44-55, 30 December, 2011
Available online at http://mije.mevlana.edu.tr

Misconception, cognitive conflict and conceptual changes in geometry: a case study with pre-service teachers

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This is a qualitative study which the participants were seven students from an Elementary Mathematics Teaching department of a Turkish University. This study primarily aimed that creating a cognitive conflict, observing their feedbacks during this period and determining how this cognitive conflict being solved. The secondary aim is recording and researching the participants' ability and behavior of using dynamic mathematics software as a cognitive tool. These participants were asked to construct a hexagon whose every sides are $\sqrt{5}$ units by using GeoGebra the dynamic mathematics software and record the whole process by Wink the screen capturing software. 15 constructions were recorded in total by 7 students. The records were uploaded to a Wiki space. The screen records were examined by the method of video analysis. Besides, the confirmation of this analysis was also done via clinical interviews. At the end of the research, it is found that students have the misconception of wrong classification of equilateral and regular polygons like in some literature knowledge. At the same time, it is observed that students misrepresented the irrational length coming from the behavior of trying to use dynamic mathematics software with an easier way. Interviewing processes clearly asserted that this type of experience helps preservice teachers to evolve their mathematical understanding and reflected on the reasons had led them to the misconception they had developed in the past.

Keyword: Misconception, Cognitive Conflict, Geometry, GeoGebra

Introduction

Misconception has been an interest of many research studies (Behr & Harel,1990; Budak & Kapusuz, 2004; Jordaan, 2005). Besides, the opportunity of learning at the existence of misconception has also been an interest, and the literature suggests that students better learn when they are allowed to make mistakes (Karadag, 2004). This suggestion set the stage for our research and led us to learning more about the conceptual change occurred after participant realization of misconception. How do participants react when they confront with their misconception? Could this confrontation be an opportunity for them to remediate their understanding? What can we, mathematics educators, learn

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from this cognitive conflict-conceptual change process in terms of improving our own educational settings and practices?

By having these questions in mind, the researchers decided to use a common geometric misconception, equilateral-regular hexagon confusion, in a dynamic mathematics environment. The research method was also structured as a demonstration of the effective use of Information and Communication Technologies (ICT) in mathematics education research. First, the partners of research team reside in different cities and even in different countries and had to communicate online throughout the research. Second, the exchange of data and information was done through the use of web 2.0 tools, particularly wiki spaces. Third, data collection was done by employing screen capturing, which is a relatively new method to collect data (Asselin & Moayeri, 2010; Cengel & Karadag, 2010; Hosein, Aczel, Clow, & Richardson, 2007; Karadag, 2009).

The following sections provide readers with the theoretical framework and methodology of the study. After discussing the findings, the paper reflects on results and our experience in this research.

Theoretical Framework

Conceptual change is different than knowledge acquisition because the former occurs if there exists a prior knowledge and the person who has misconceived this certain type of knowledge is aware of his or her misconception and is willing to evolve his or her understanding towards a correct conceptualization. However, knowledge acquisition needs a lack of prior knowledge – missing knowledge –and/or an existence of incomplete knowledge – gap in knowledge (Chi, 2008). Regardless of the type of conceptual change –whether within or across ontological categories (Chi, 1992), it is quite safe to claim that the first condition to remove misconception is to make the person realize the ill-structure that she or he has. However, realization is necessary but not satisfactory, because the person, then, needs to be convinced to replace the current misconception with the correct one. In other words, "the major goal is to create a cognitive conflict to make the learner dissatisfied with his or her existing conception" (Ozdemir & Clark, 2007, p. 352).

Chi (1992) discriminates the conceptual change within a category and the one across categories and argues that the change in the latter is almost a knowledge acquisition process because the ontological description of concept changes in this case. For example, considering geometric figures such as polygons and circles as functions or relations –that is, analytic form– demands completely different type of thinking than what is done in geometry courses, and therefore, leads to the acquisition of a new set of knowledge. Since the conceptual change across categories is beyond the scope of the research being reported here, we will devote our attention to conceptual change within categories.

Figure 1 depicts a tree diagram to represent subcategories of polygon concept within its own category and to delineate the misconception that students may have. As seen in the figure, hexagons can be categorized whether their sides are equal or not. Furthermore, equilateral hexagons can be classified with respect to the regularity property. Regular hexagons form a branch of the tree of equilateral hexagons because they have some special constraints. That is, their interior –also exterior –angles at each corner have to be equal whereas this constrain is not a necessary condition for equilateral hexagons. This is the point where many people may have developed a misconception, as a concept in conflict with the correct one but constructed in one's mind as if it is the correct concept (Yenilmez &Yasa, 2008).

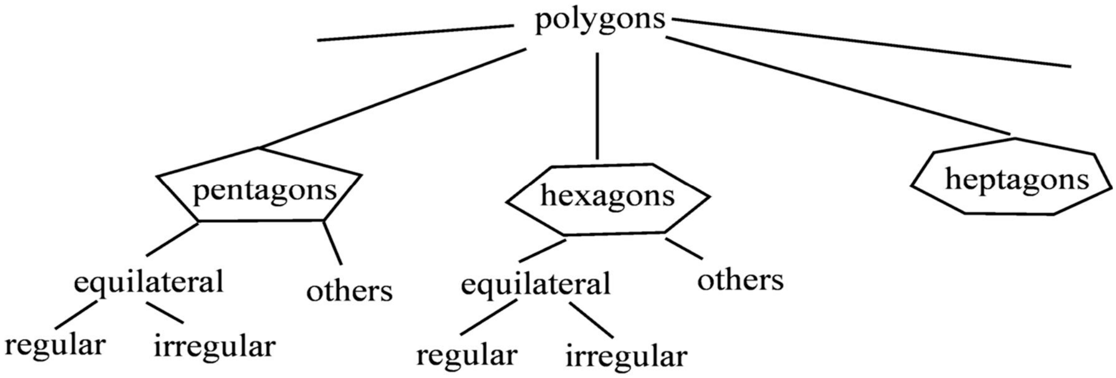

Figure 1. Subcategories of polygon

A similar misconception about regular pentagons has been identified by Ubuz (1999). She stated that students tend to apply the features of regular pentagons to any type of pentagon. It appears that students use the terms of regular and equilateral (polygon) interchangeably without paying attention and/or knowing the difference between them. Prior to describing our strategy to remedial this ill structure, it seems to be a good idea to review the framework describing conceptual change for similar cases. Chi (1992) asserts three types of conceptual change within a category, which serves for our purpose as well, to alter misconceptions.

Revision of part-whole relationships

Trapezoids can be defined as quadrilaterals having two opposite sides parallel to each other while parallelograms can be considered as quadrilaterals having also the other opposite sides parallel in addition to the property trapezoids have. Thus, one can interpret the relationships between these two concepts as one of the cases illustrated in Figure 2.

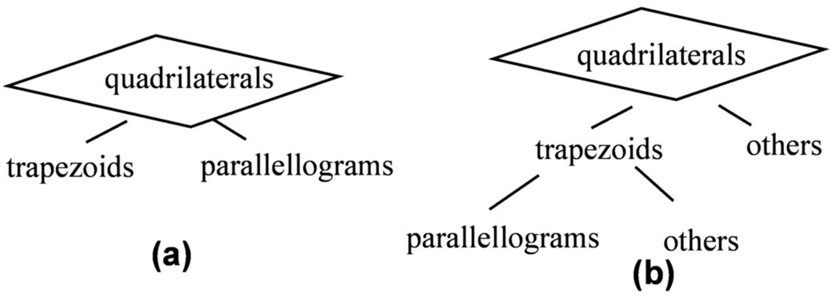

Figure 2. Reorganization of the concept of Quadrilaterals

Moving from (a) to (b) demands a reorganization of the concepts, because no new knowledge is acquired. Also, the change could be achieved through a careful discussion of features.

Formation of new categories

Referring to the example we used in our research, regular hexagons have equal angles whereas equilateral hexagons may have unequal angles too (see Figure 3). The change occurs here is actually the result of a cognitive process because participants are assumed to differentiate one group of figures – regular hexagons –from others –irregular hexagons –to form a new branch under the same tree – equilateral hexagons.

Therefore, the conceptual change in this particular case could be considered as an acquisition process rather than a straightforward migration of subcategories. Aforementioned cognitive processes are the kind of differentiation and integration as well as the generalization of subcategories because the person should put a cognitive effort to differentiate the current concepts and integrate in a different way to establish the new cognitive schema.

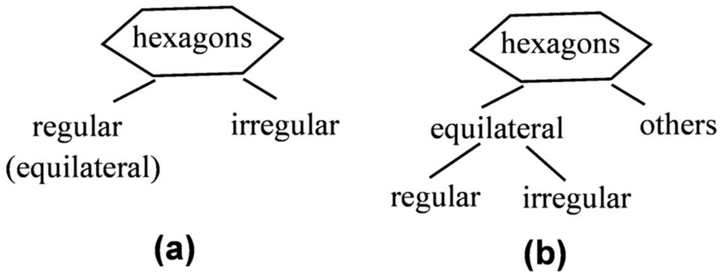

Figure 3. Reorganization of the concept of Hexagons

Reclassification of existing categories

Considering a reclassification of figure 1 as illustrated in the figure 4 requires the formation of new categorical structure. This type of transformation –change– from one existing categorical structure to another one can be achieved without a migration of concepts and even without a really substantial cognitive effort to re-structure his or her understanding but re-classify the current understanding.

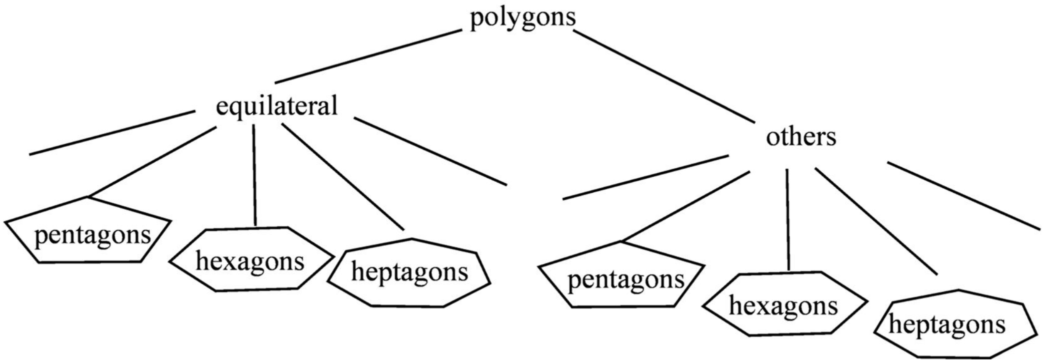

Figure 4. Reclassification of Figure 1

In this research, we aimed to identify the existence of misconceptions in preservice teachers' understanding of equilateral hexagons and to document the conceptual change as a process. Besides reporting conceptual change as a complete process starting from misconception to arriving at the replacement of correct information with the one in conflict, we also aimed to document participants' use of GeoGebra as dynamic mathematics software, which provides us with significant information on how they had instrumentalized (Artigue, 2002) this software as a cognitive tool. Regarding the use of technology, this is also an empirically tractable question for us because we also concern about the way how technology is integrated in mathematics education.

Methodology

In this qualitative research, 7 pre-service teachers studying mathematics education at a Turkish University were recruited for the study and asked to construct an equilateral hexagon whose sides are

$\sqrt{5}$ units in length. Although it was explicitly emphasized them to create equilateral hexagons, our assumption was that they, at least some of them, would create regular hexagons instead.

They used GeoGebra, which is a dynamic and interactive mathematics learning environment (DIMLE) –a term coined by Martinovic and Karadag (2011)-, to construct their artifacts and Wink to record their work. They were asked to talk aloud while constructing their artifacts such that Wink could record their talking. After completing their work, they submitted their work at a public wiki space because the communications between researchers as well as between participants and researchers have been performed online.

Their work was analyzed by employing video analysis techniques. Then, selected students were interviewed to validate our interpretation of the results and to elaborate on the results of analyses. During the interviews, students were presented their own construction(s) as well as the constructs of equilateral but regular hexagons, developed by researchers.

Findings

The number of constructs submitted to the wiki space by 7 preservice teachers was 15 in total, because some participants submitted more than one construct. The main reason to submit more than one solution is that because those participants intended to illustrate their knowledge to be able to create different constructions. The difference in constructions was rooted in various ways. For example, one of the participants used three different techniques such as (1) 1-2-$\sqrt{5}$ right angle triangle, (2) latex command, and (3) analytic formula of circle to make GeoGebra calculate and create a line segment whose length is $\sqrt{5}$. Similarly, they used various techniques to construct the hexagon, such as constructing (1) by using circles, (2) by reflecting triangles, or (3) by marking end points of line segments and connecting them.

In order to investigate how much participants have instrumentalizedGeoGebra, screen capturing as a data collection method seems to be an effective strategy because we could have a chance to analyze every second of their construction processes. This analysis has revealed that participants have developed in varying degrees of competencies of instrumentalization. The detail of the following description of their construction processes is provided in Appendix.

Majority of participants preferred keeping grid view and axes active although only a few of them used gridlines in their constructions. Their way of using gridlines was usually to construct a line segment, whose length is $\sqrt{5}$, by using two adjacent grid squares, that is a rectangle whose sides are 1 and 2 and therefore whose hypotenuse is $\sqrt{5}$.

Regarding drawing a line segment whose length is $\sqrt{5}$, two students used sqrt command while one of them preferred using latex command in their constructions. Also, four of seven students used GeoGebra regular polygon tool to construct hexagon in the six constructions out of fifteen. Therefore, their preference in using GeoGebra tool provides enough evidence for us to assume that their intention was to construct regular hexagons.

Other participants who were not that explicit in using GeoGebra regular polygon tool demonstrated their impressive knowledge of the features of regular hexagons while creating their artifacts. For example, their way of using triangles equilateral triangles, circles, and angles convinced us about their content knowledge and the misconception about the distinction between regular and equilateral hexagons.

More specifically, figure 5 illustrates one construction using 1-2-$\sqrt{5}$ right angle triangle to draw the line segment with a length of $\sqrt{5}$ and using GeoGebra angle command to draw interior angles whose measures are 120 degree each. This construction explicitly illustrates student misconception while drawing an equilateral hexagon because her intention, as seen in her construction, is to create a regular hexagon, whose interior angles are 120 degree. We, researchers, interpret this intention as an evidence of her misconception. Furthermore, one of the researchers, who collected data and is the first author of this paper, explicitly asked her intention in the interview to confirm and validate our interpretation.

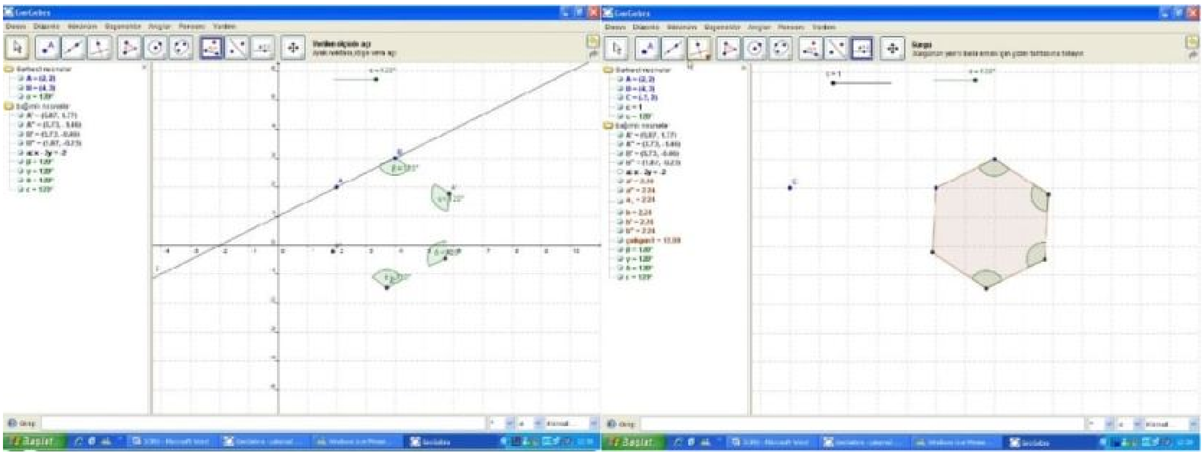

Figure 5. Student construction with GeoGebra

Similarly, figure 6 illustrates another example constructed by the same participant. In this construction, the participant creates an equilateral triangle and reflects the triangle over one of its sides to have the second identical triangle created. Again, our analysis of complete construction process illustrated another construction of regular hexagon. In fact, constructing an equilateral triangle as a starting point seems a strong evidence for the existence of misconception because only regular hexagons have equilateral triangles as part of their internal structures.

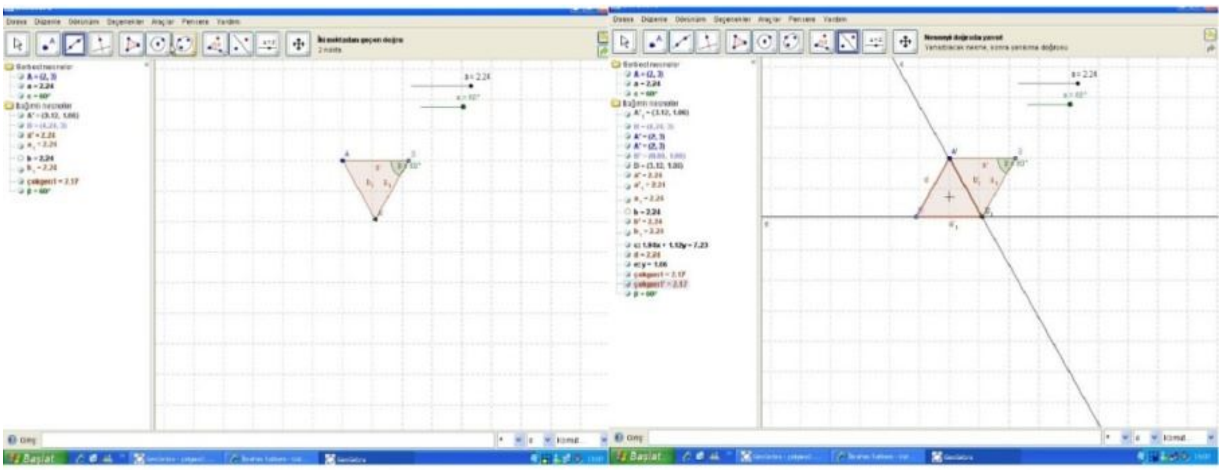

Figure 6.Student construction with GeoGebra-II

In sum, all of the 15 constructions from 7 students explicitly or implicitly demonstrated that participants had a misconception in distinguishing equilateral and regular hexagons. Then, we interviewed students to confront with their misconception and to experience a cognitive conflict. Transcription of interview data clearly illustrates how they confronted with the conflict and how they replaced their misconception with the correct one.

The interview, conducted by one of the participants and represented by a code name Berke, may make clearer that how the general misconception was inferred and the conceptual change process. Berke created one of the impressive approaches to construct the equilateral hexagon. He constructed a line segment with the length of $\sqrt{45}$ by connecting the points (-3, 0) and (0, 6) then used the regular polygon tool of GeoGebra to construct a equilateral triangle. Last, he reflected this equilateral triangle by the intersection point of the medians. These two triangles' intersection points on sides divided the sides into three equal parts and constituted the corners of the hexagon (Figure-7).

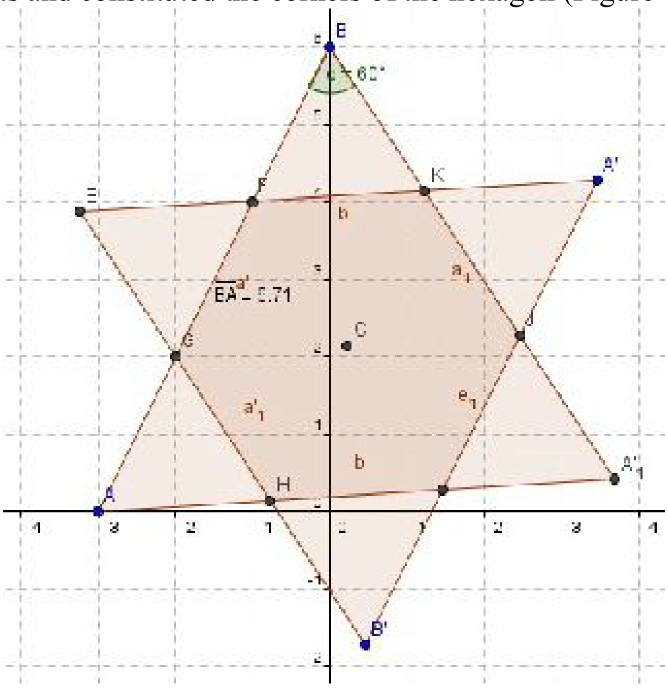

Figure 7. Berke's construction with GeoGebra

Even if Berke showed a great ability to obtain the length of $\sqrt{5}$ and create an equilateral hexagon, his construction is also regular as all other's constructions. Let's look the remaining part of the interview;

…

Researcher: Do you remember the original version of the task?

Berke: Yes! We were asked to construct a hexagon that any of its sides is $\sqrt{5}$.

It is understood that Berke is aware of the original task.

Researcher: I will show three different constructions. Please observe them and decide that any of these can be a true solution or not. First one is.

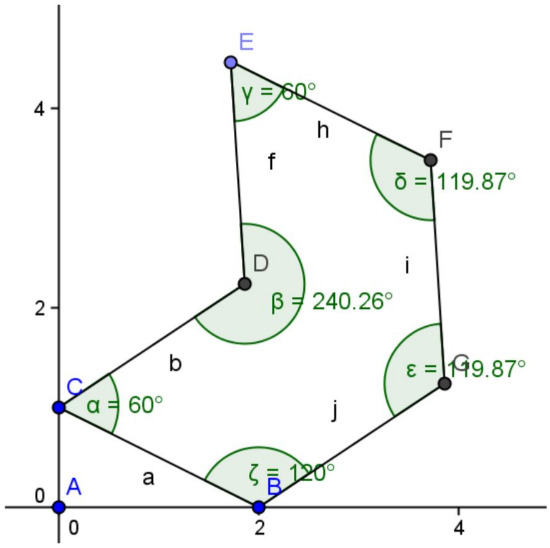

Figure 8. First construction presented to participants.

Berke: I can say "No!" Yeah this can not be true.

Berke certainly disagree on this construction is one of the right construction even if it is.

Researcher: OK! Take a look this one.

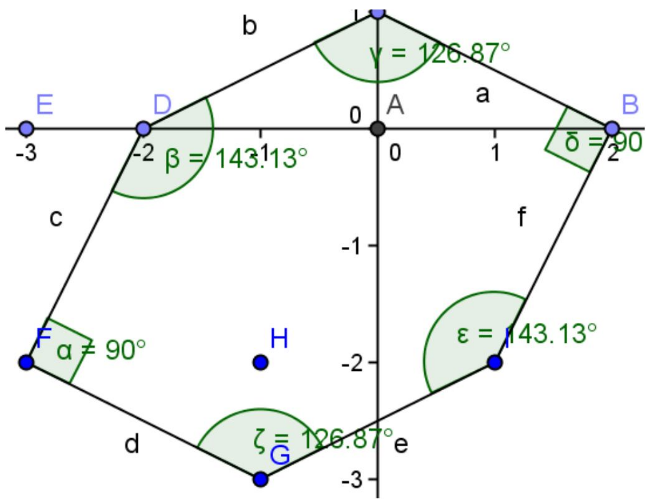

Figure 9. Second construction presented to participants.

Berke: This one maybe true!

We have no an explicit evidence that why Berke assumes this construction may be true. The previous one was a concave polygon and extraordinary according to this construction.

Researcher: You are not still sure! Observe the last!

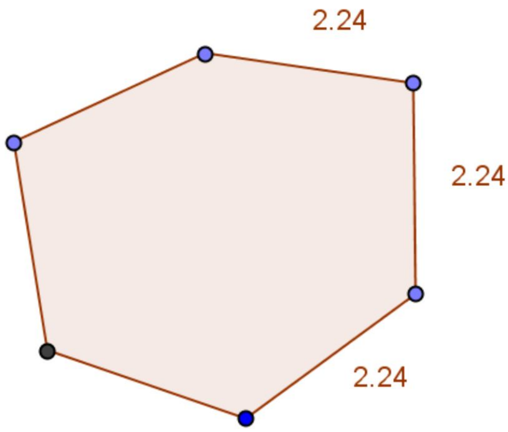

Figure 9. Third construction presented to participants.

Berke: This must be true

He was confident when the lengths are shown. After showing why the first two constructions are also right, he is asked to explain the difference between his own construction and these ones.

Researcher: What is the difference between your construction and these?

Berke: Especially first two constructions are not regular hexagon even if they are equilateral.

Researcher: Could you define the regular hexagon.

Berke: The hexagon which has equal sides and one interior angle is 120 degrees.

Researcher: What was your construction?

Berke: A regular hexagon

Researcher: What was the question?

Berke: "Construct a regular hexagon with the side length of $\sqrt{5}$ *".*

Resercher: No! The task was not like that! You had already stated the task as "construct a hexagon that any of its sides is $\sqrt{5}$ *" just before.*

Berke was surprised after understanding that he implicitly focused on constructing a regular hexagon. He also admit some reasons why he and other 6 students though same way.

Berke: We used to see regular polygons because of the OSS (Turkish name of the multiple choice university entrance exam). Almost all of our calculations are related to regular shapes. So we have this scheme subconsciously. It must be important to ask students to construct their own scheme by using definitions.

Discussion

This study reports on a remedial session followed by self-reflection of pre-service teachers. In that sense, the study itself seems a good example of turning misconception to a learning experience. With the supportive guidance of interviewer, students not only remediate their own misconception but also benefit from this experience to reflect on their own pedagogical content knowledge, which is an important practice for pre-service teachers.

The data reveals that all participants had the same misconception although we had assumed that most of the participants will have misconception about the aforementioned example as previously stated by Ubuz (1999). Moreover, a second possible misconception also appeared by the use of dynamic software in this study. When students met a situation that they have to obtain the length $\sqrt{5}$, they may see both construction of this length and using some commands to obtain the number in software as equivalent. Whereas using any latex command to obtain an irrational number is just an approximation. Only the construction, by using 1-2-$\sqrt{5}$ triangle, is the real value of an irrational number. This may come from a desire of using the software in an easier way. Even if only 3 of 7 students revealed an explicit clue on preferring to use easy commands to obtain the number $\sqrt{5}$, this is an important issue for us to prevent new misconceptions coming from the use of technology, also an opportunity that can be assessed to review the meaning of irrational number.

One of the most important and valuable result that can be extracted from this study is not only the existence of misconception nor the method we employed to make it explicit. Rather, the most exciting result is to document how participants reflect on their previous conceptualization periods and how they have started seeking and suggesting strategies to prevent this type of misconception.

To sum, this study shows that this particular type of cognitive conflict –and also cognitive change as an outcome – is quite robust not because of mathematics, but rather because of pedagogical consequences of experiment. Further evidence we have gathered through interviewing processes clearly asserts that this type of experience helps preservice teachers to evolve their understanding of mathematical content and, even more importantly, to reflect on the reasons had led them to the misconception they had developed in the past. Moreover, this experience encourages them to seek possible strategies to avoid unwanted misconception formation.

Acknowledgements

Another version of this paper was presented IETC 2011 as an oral presentation

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Appendix A

Initial Analysis of Data

Summary

Where: a Turkish University
When: November 2010
How many students: 7 students and 15 constructions

Student name (Gender)	Construction number	Grid view	Axes	Calculating $\sqrt{5}$	Construction method	Using GeoGebra “regular polygon” tool	Sound recording	Stating “regular…”	Using theoretical knowledge leading to “regular...”
BA (F)	1	yes	yes	construction	End points	no	no	no	yes
	2	yes	yes	Latex command	Reflected triangles	no	no	no	yes
	3	yes	yes	Circle radius	Three-circle construction	no	No	no	yes
	4	yes	yes	Circle radius	Six circle	no	no	no	yes
EE (M)	1	no	yes	construction	Reflected triangles	no	no	no	yes
ErE(M)	1	yes	yes	construction	GeoGebra tool	yes	no	no	yes
	2	yes	yes	construction	GeoGebra tool	yes	no	no	yes
FK (F)	1	yes	no	construction	GeoGebra tool	yes	yes	yes	yes
	2	no	yes	construction	Equilateral triangles	no	yes	no	yes
GK (F)	1	yes	yes	construction	Equilateral triangles	no	no	no	yes
	2	yes	yes	construction	GeoGebra tool	yes	no	no	yes
	3	yes	yes	construction	GeoGebra tool	no	no	no	yes
HP (M)	1	no	yes	Sqrt command	End points	no	yes	yes	yes
SS (F)	1	no	yes	Sqrt command	GeoGebra tool	yes	yes	yes	yes
	2	yes	yes	Sqrt command	End points	no	yes	yes	yes