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Implications of unequal rates of population growth for trade: An overlapping generations-general equilibrium analysis within the Heckscher-Ohlin framework*

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Abstract

This paper considers a two-country world where countries have unequal population growth rates, and presents closed-form solutions to a two-sector overlapping generations model to explore possible implications of differential population dynamics for trade. The countries are characterized by identical preferences and production technologies as in the static Heckscher-Ohlin (HO) framework. Our results reveal that differences in population growth rates will create comparative advantages in the same way as suggested by the static HO model, but contrary to the predictions of this model, free trade will not necessarily imply welfare gains for both parties.

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1. Introduction

The standard 2-commodity, 2-factor and 2-country (2x2x2) Heckscher-Ohlin (HO) model of international trade considers two countries that have identical preferences and production technologies but different capital-labor ratios and shows that this difference in relative factor endowments would be sufficient for trade to be Pareto-superior to autarky, provided that the factor intensities of production differ across commodities –see, for example, the nicely written chapter on the HO model in Salvatore (2001) for additional assumptions and a full description of the model.

While very useful in a static context, this model does not offer much insight into the dynamics of trade or gains from trade over time. In fact, HO type incentives for trade would eventually disappear, since trade itself would lead to an elimination of the initial differences between relative factor endowments, if countries were to be identical in every other respect (Chen, 1992). One may then argue that for trade to continue over time, additional differences such as unequal saving/time preference or population growth rates would be needed to make factor proportions evolve continuously.

In the 1960s and 1970s, various authors extended multi-sector growth models into the standard, two-country set-up of the HO model to study the nature of dynamic trade equilibrium by considering exogenous differences in savings and population growth rates of nations as determinants of comparative advantages. Oniki and Uzawa (1965) used the two-sector growth model in a two-country setting to study the effects of capital accumulation and labor force growth on international equilibrium over time. Findlay (1970) added a non-traded capital good sector to the Oniki-Uzawa model and studied the behavior of relative factor endowments, by considering a small country facing a fixed world terms of trade. He concluded that the long-run pattern of comparative advantage depends ultimately on the propensity to save and the growth rate of labor force. While many similar studies overlooked the question of what caused differences in saving rates themselves, Stiglitz (1970) suggested that the differences in saving rates could be explained by differences in time preference rates and hence could serve as long-run determinants of comparative advantages. Much later in 1997, Galor and Lin used a similar line of reasoning while describing dynamic micro foundations of the HO model on the basis of differences in time preference rates across two nations, but this time within the context of an overlapping generations general equilibrium (OLG-GE) model. Yet, it is arguable, as Chen

(1992) suggests, that consideration of differences in time preference rates would truly be in the spirit of the HO model since the standard formulation of this model assumes away differences in preferences and production technologies so as to highlight the role of differences in factor endowments.

In this paper, we focus on disparities between population growth rates of trading nations instead and study their role as *á la* Heckscher-Ohlin determinants of dynamic comparative advantages based on closed-form solutions to an OLG-GE model as in Galor and Lin (1997). We also investigate welfare consequences of trade as in Cremers (2005) but differently than the Cremers study, we consider areas whose populations grow at differing speeds and investigate whether welfare predictions of the static HO model would remain valid.

The effects of the differential speed of demographic transition observed across nations and the resulting differences in population growth rates are interesting to study as they become increasingly more relevant as a determinant of global trade patterns. The price setting power that (the People's Republic of) China already began to exercise in international markets is to a very large extent a result of its current position as the most populous country in the world with a predominantly young population. Given the recent projections on global population trends by the United Nations, the empirical relevance of population growth disparities as a determinant of trade in the decades ahead should be expected to increase even further. By these projections, the observed gap between the population growth rates in the labor-abundant "South" and the capital-abundant "North" will gradually diminish over time, but is not likely to disappear even after the year 2050. Despite this rapidly increasing relevance, the recent dynamic trade literature is just re-discovering the significance of changes in relative factor endowments induced by the differential speed of demographic transition in these two areas, after the explorations carried out by using multi-sector growth models of the 1960s and the 1970s –see Sayan (2002) for an extended review of the literature.

Even though these multi-sector growth models are capable of indicating the directions and magnitude of changes in sectoral trade flows in response to changes in relative endowments of nations with different population growth rates,[1] they are not suitable for showing the evolution of these endowments by allowing for differences in the age compositions of populations which would naturally arise when populations grow at differing speeds. To address these, an overlapping generations, general equilibrium structure is needed. Such a structure would implicitly capture changes in the saving behavior of individuals over the working and retirement phases of their life cycles, thereby letting relative factor endowments to evolve, due not only to the changes in labor supply, but also to the changes in capital accumulation resulting from the adjustments of savings to the changing age profile of population (Kenç and Sayan, 2001; Sayan, 2005). Yet, there is a severe lack of studies in the literature studying the nature of long-run trade equilibrium by extending the HO model into a dynamic, overlapping generations framework with non-stationary populations.

In order to contribute to the filling of this gap in the literature, we consider a world that is made up of two economies each producing two commodities, by using two factors of production and are identical in every other respect than the population growth rates. This enables us to focus on the long-run autarky equilibrium of one economy, and project the results on the sensitivity of steady state values of endogenous variables to changes in the population growth rate into the trade set-up. The economies we consider are populated by two overlapping generations of individuals each living for two periods, and the population in each country is allowed to grow at a constant rate that is different than the other.[2]

While our results establish that population growth rate differences will serve as determinants of HO-type comparative advantages in a dynamic set-up, they also reveal that free trade will not necessarily imply welfare gains for both parties unlike what the static HO model would predict. Thus, our model extends the

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[1] By contrast, Deardorff (1987) used a one-sector, two-country growth model to investigate what may happen to relative factor endowments over time, if population in one of the countries grows at a higher rate than in the other. Since this one-sector growth model is not suitable for analyzing commodity composition of trade, however, Deardorff is forced to infer what would happen to trade by linking changes in factor proportions over time to static HO results from a previously developed model with many goods (Deardorff, 1979).

[2] Sayan (2005) also considers the case population of growth rates declining at different speeds over time through a simulation experiment and shows that the direction of results would not change depending on whether disparities between population growth rates remain the same or get eliminated gradually.

previously studied OLG set-ups with stationary populations where trade may not improve welfare for both parties (Fried 1980, Galor 1988, Mountford 1998, Cremers 2005). Furthermore, the results from closed-form solutions reported here are completely consistent with those recently obtained from numerical solutions of a variant of the present model in Sayan (2005).

The rest of the discussion is organized as follows. The next section describes the model. Section 3 presents the long-run closed-form solutions, and discusses the implications of population growth differences for trade. Section 4 focuses on welfare effects of trade. Section 5 concludes the paper.

2. The Model

We start by assuming that a generation made up of N_t^i individuals is born in each area i ($i = N, S$) at every period t and the population grows at the rate of n^i ($-1 < n^i$) so that $N_t^i = (1+n^i)N_{t-1}^i$. Here, N represents the North, whereas S represents the South such that $n^N < n^S$.

For all periods t, individuals born and living the first period of their lives at time t (i.e., the youngs) work and save a fraction of their labor income to finance their old age consumption after they retire in the next period. These old individuals do not save (consuming their entire savings from the previous period) in the second and last period of their lives since the model assumes away bequests and similar intergenerational transfers.

At any period *t*, the ratio of youngs to total population made up of the youngs born in *t* and the olds born in *t-1* in each area will be given by $\frac{N_t^i}{N_{t-1}^i + N_t^i} = \frac{(1+n^i)N_{t-1}^i}{[1+(1+n^i)]N_{t-1}^i} = \frac{(1+n^i)}{(2+n^i)}$ such that $\frac{(1+n^S)}{(2+n^S)} > \frac{(1+n^N)}{(2+n^N)}$ $\forall t$. This implies that the South always has a higher population share of young working individuals than the North.

Given the way results for individual countries are compared in the sections ahead, superscripts N and S will be dropped not to clog the notation and the discussion will be carried out in reference to a single country unless specified otherwise.

2.1. Consumption and saving

For all periods t, the youngs born at time t inelastically supply a fixed amount of labor, $\bar{l}$, earn labor income at the competitive wage rate, w_t, and decide on how to allocate it between first period consumption of goods 1 and 2 $\left(c_{1yt}, c_{2yt}\right)$, and savings, s_t, which bring interest earnings at the rate of r_{t+1} in the next period. In the second period, they retire and consume c_{1ot+1} units of good 1, and c_{2ot+1} units of good 2 by spending all their capital income from the previous period's savings. Good 1 is assumed to simultaneously serve as a consumption and investment good, whereas Good 2 is assumed to be demanded for consumption purposes alone.[3] Given the price, p_t, of the consumption good (good 2) in terms of the investment-consumption good (good 1) at time t, each individual solves the following problem:

$$\max\left(c_{1yt}^{\theta} c_{2yt}^{1-\theta}\right)^{\mu}\left(c_{1ot+1}^{\theta} c_{2ot+1}^{1-\theta}\right)^{1-\mu}$$

subject to

$$c_{1yt} + p_t c_{2yt} + \frac{1}{1+r_{t+1}}\left(c_{1ot+1} + p_{t+1} c_{2ot+1}\right) = w_t \bar{l}$$
$$c_{1yt}, c_{2yt}, c_{1ot+1}, c_{2ot+1} \geq 0 \tag{1}$$

where $0<\theta<1$, $0<\mu<1$.

The solution to this problem results in the following consumption decisions:

$$c_{1yt} = \mu\theta w_t \bar{l} \tag{2}$$

$$c_{2yt} = \mu(1-\theta)\frac{w_t \bar{l}}{p_t} \tag{3}$$

$$c_{1ot+1} = (1-\mu)\theta(1+r_{t+1}) w_t \bar{l} \tag{4}$$

$$c_{2ot+1} = (1-\mu)(1-\theta)(1+r_{t+1})\frac{w_t \bar{l}}{p_{t+1}}, \tag{5}$$

[3] This classification of goods is different than what has been assumed in Galor (1992) where each sector is assumed to produce either a consumption good or an investment good.

2.2. Production

Both the investment-consumption good and the consumption good are produced according to constant returns to scale Cobb-Douglas production technologies by using capital, K, and labor, L. Sectoral outputs X_{1t} and X_{2t} are given in per capita terms by $x_{1t} = k_{1t}^{\alpha} l_{1t}^{1-\alpha}$ and $x_{2t} = k_{2t}^{\beta} l_{2t}^{1-\beta}$ (for $0 < \alpha < 1, 0 < \beta < 1$) where $x_{it} = \frac{X_{it}}{N_t}$, $k_{it} = \frac{K_{it}}{N_t}$ and $l_{it} = \frac{L_{it}}{N_t}$ for $i = 1,2$. We assume following Galor (1992) that production of investment-consumption good is relatively more capital intensive and hence let $\alpha > \beta$.

Total labor supply at time t, $L_t = N_t \bar{l}$ and factor market equilibrium requires that $k_{1t} + k_{2t} = k_t$ and $l_{1t} + l_{2t} = \bar{l}$. The demands for labor and capital in each sector are characterized by the first order conditions for profit maximization. If labor and capital are perfectly mobile across sectors and if both goods are produced, then $r_t = \alpha k_{1t}^{\alpha-1} l_{1t}^{1-\alpha} = p_t \beta k_{2t}^{\beta-1} l_{2t}^{1-\beta}$, and $w_t = (1-\alpha) k_{1t}^{\alpha} l_{1t}^{-\alpha} = p_t (1-\beta) k_{2t}^{\beta} l_{2t}^{-\beta}$.

$\bar{l}$ can be normalized to 1 without loss of generality. Then, the solution of the producers' problem gives

$$l_{1t} = \frac{\delta}{\delta - \varepsilon} - \frac{1}{\delta - \varepsilon} k_t p_t^{\frac{1}{\beta-\alpha}}, \tag{6}$$

$$l_{2t} = -\frac{\varepsilon}{\delta - \varepsilon} + \frac{1}{\delta - \varepsilon} k_t p_t^{\frac{1}{\beta-\alpha}}, \tag{7}$$

$$k_{1t} = -\frac{\varepsilon}{\delta - \varepsilon} k_t + \frac{\delta\varepsilon}{\delta - \varepsilon} p_t^{\frac{1}{\alpha-\beta}}, \tag{8}$$

$$k_{2t} = -\frac{\delta}{\delta - \varepsilon} k_t - \frac{\delta\varepsilon}{\delta - \varepsilon} p_t^{\frac{1}{\alpha-\beta}}, \tag{9}$$

where

$$\varepsilon = \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{1-\beta}{\alpha-\beta}}, \tag{10}$$

$$\delta = \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha-\beta}}. \tag{11}$$

Hence,

$$r_t = \alpha\varepsilon^{\alpha-1} p_t^{\frac{\alpha-1}{\alpha-\beta}} = \beta\delta^{\beta-1} p_t^{\frac{\alpha-1}{\alpha-\beta}}, \tag{12}$$

$$w_t = (1-\alpha)\varepsilon^{\alpha} p_t^{\frac{\alpha}{\alpha-\beta}} = (1-\beta)\delta^{\beta} p_t^{\frac{\alpha}{\alpha-\beta}}, \tag{13}$$

and per capita outputs can now be written as $x_{1t} = l_{1t}\varepsilon^{\alpha} p_t^{\frac{\alpha}{\alpha-\beta}}$, and $x_{2t} = l_{2t}\delta^{\beta} p_t^{\frac{\beta}{\alpha-\beta}}$.

2.3. The autarky equilibrium

A perfect-foresight autarky equilibrium can now be viewed as a collection of $\{k_t, p_t\}_{t=0}^{\infty}$ pairs clearing the goods markets at every period t, while satisfying the dynamics of the capital stock at time $t+1$. The fraction of income saved during the first period of life is $(1-\mu)$. Thus, the evolution of per capita capital is governed by

$$k_{t+1} = \frac{(1-\mu)w_t}{(1+n)} \tag{14}$$

The clearance of the goods' market in period t requires that the per capita supply of each good be equal to its respective per capita demand. Hence,

$$x_{1t} + k_t = c_{1yt} + \frac{1}{(1+n)}c_{1ot} + (1+n)k_{t+1} \tag{15}$$

$$x_{2t} = c_{2yt} + \frac{1}{(1+n)}c_{2ot} \tag{16}$$

Since Walras' law holds, we can focus on the market clearance condition for the consumption good (good 2) alone. Substituting for c_{2yt} and c_{2ot} from (3) and (5), using (7), (12), (13), and remembering that $x_{2t} = l_{2t}\delta^{\beta} p_t^{\frac{\beta}{\alpha-\beta}}$, one obtains

$$k_t = \phi_1 p_t^{\frac{1}{\alpha-\beta}} + \phi_2 p_{t-1}^{\frac{\alpha}{\alpha-\beta}} p_t^{\frac{1-\alpha}{\alpha-\beta}} + \phi_3 p_{t-1}^{\frac{\alpha}{\alpha-\beta}}, \tag{17}$$

where

$$\phi_1 = \mu(1-\theta)(1-\beta)(\delta-\varepsilon) + \varepsilon, \tag{18}$$

$$\phi_2 = \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)(\delta-\varepsilon), \tag{19}$$

$$\phi_3 = \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)(\delta-\varepsilon)\beta\delta^{\beta-1}. \tag{20}$$

Now, substituting (13) into (14), per capita capital dynamics equation can simply be written as

$$k_{t+1} = \phi_4 p_t^{\frac{\alpha}{\alpha-\beta}}, \tag{21}$$

where

$$\phi_4 = \frac{1}{1+n}(1-\mu)(1-\beta)\delta^{\beta}. \tag{22}$$

Remembering (17) and using (21) one can obtain a nonlinear difference equation in terms of price ratios only. This equation characterizes the dynamics of our model economy and is given by

$$(\phi_4 - \phi_3) p_t^{\frac{\alpha}{\alpha-\beta}} = \phi_1 p_{t+1}^{\frac{1}{\alpha-\beta}} + \phi_2 p_t^{\frac{\alpha}{\alpha-\beta}} p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}}. \tag{23}$$

3. Long-run closed form solutions under Autarky

3.1. Steady state values of key variables

The steady state value of p_s satisfies (23) with $p_{t+1} = p_t = p_s$. Ruling out $p_s = 0$, p_s under autarky is given by

$$p_s = \Phi^{\frac{\alpha-\beta}{1-\alpha}}, \quad where \quad \Phi = \frac{\phi_4 - \phi_3}{\phi_1 + \phi_2}. \tag{24}$$

Proposition

The equilibrium price ratio, p_s, for this perfect foresight overlapping-generations general equilibrium model with constant returns to scale production exists and is unique for all values of $-1 < n$ and given values of α, β, μ, θ that lie strictly between 0 and 1 such that $\alpha > \beta$.

Proof

Straightforward: Uniqueness follows from the closed form solution for p_s in (24), and the existence is assured by the fact that $\Phi > 0$ for the given parameters.

Consequently, the closed form solutions for the steady state per capita values are obtained as

$$k_s = \phi_4 \Phi^{\frac{\alpha}{1-\alpha}}, \tag{25}$$

$$w_s = (1-\alpha)\varepsilon^{\alpha}\Phi^{\frac{\alpha}{1-\alpha}}, \tag{26}$$

$$r_s = \alpha\varepsilon^{\alpha-1}\frac{1}{\Phi}, \tag{27}$$

$$c_{1ys} = \mu\theta(1-\alpha)\varepsilon^{\alpha}\Phi^{\frac{\alpha}{1-\alpha}}, \tag{28}$$

$$c_{2ys} = \mu(1-\theta)(1-\alpha)\varepsilon^{\alpha}\Phi^{\frac{\beta}{1-\alpha}}, \tag{29}$$

$$c_{1os} = (1-\mu)\theta(1-\alpha)\varepsilon^{\alpha}\left(1+\alpha\varepsilon^{\alpha-1}\frac{1}{\Phi}\right)\Phi^{\frac{\alpha}{1-\alpha}}, \tag{30}$$

$$c_{2os} = (1-\mu)(1-\theta)(1-\alpha)\varepsilon^{\alpha}\left(1+\alpha\varepsilon^{\alpha-1}\frac{1}{\Phi}\right)\Phi^{\frac{\beta}{1-\alpha}}, \tag{31}$$

Since the economies we consider are assumed to be identical in every respect but their population growth rates, steady state values for all variables in (24) through (31) will be the same except for differences in population growth rates entering into equations (19)-(20) and (22). Then, the equality of population growth rates would leave no room for trade, but as long as $n^N < n^S$, relative commodity and factor prices that prevailed under autarky will be differentiated creating incentives for trade in the same way as suggested by the static HO model.

Equations (25) through (31) can now be used to evaluate the effects of population growth rate differences on relative values of steady state autarky equilibrium in N and S, enabling us to make inferences about the direction and consequences of trade.

Corollary 1

The steady state value of per capita capital decreases in n.

Given (25),

$$\frac{\partial k_s}{\partial n} = \Phi^{\frac{\alpha}{1-\alpha}}\frac{\partial \phi_4}{\partial n} + \phi_4\left(\frac{\alpha}{1-\alpha}\right)\Phi^{\frac{\alpha}{1-\alpha}-1}\frac{\partial \Phi}{\partial n}. \tag{32}$$

Since $\phi_4 > 0$, $\Phi > 0$ and $\frac{\partial \phi_4}{\partial n} = -\frac{\phi_4}{1+n} < 0$, and $\frac{\partial \Phi}{\partial n} < 0$, $\frac{\partial k_s}{\partial n} < 0$. Thus, the economy with the lower (higher) population growth rate would have the higher (lower) stock of capital per capita under autarky. This establishes N as the relatively capital-abundant region and S as the relatively labor-abundant region.

In the light of Corollary 1 and the predictions of the static HO model, one would expect N to have a comparative advantage in the production of capital-intensive commodity (investment-consumption good) and to export it, whereas S is expected to have a comparative advantage in the production of labor-intensive commodity that serves as a consumption good alone. To see if this will indeed be the case, the next section investigates the effects of the assumed difference in population growth rates on initial autarky prices so as to identify the direction of commodity flows between the labor-abundant South and the capital-abundant North after the opening of trade.

3.2. Differences in n and comparative advantages

Corollary 2

The equilibrium relative price ratio, p_s, is decreasing in the population growth rate n.

The effect of the population growth rate, n, on the steady state price ratio is given by

$$\frac{\partial p_s}{\partial n} = \left(\frac{\alpha - \beta}{1-\alpha}\right)\Phi^{\frac{\alpha-\beta}{1-\alpha}-1}\frac{\partial \Phi}{\partial n}. \tag{33}$$

Since $\frac{\partial \Phi}{\partial n} < 0$,

$$\frac{\partial p_s}{\partial n} < 0 \qquad \textit{for} \quad \alpha > \beta. \tag{34}$$

Thus, the equilibrium relative price of a good decreases with n, if the production of that good is relatively labor-intensive. Remembering that $n^S > n^N$, this implies that the area with the rapidly growing population will have a relative cost advantage in the production of labor-intensive commodity, whereas the country with the slowly growing population will have a relative cost advantage in the production of capital-intensive commodity.

To summarize, given two areas such as N and S that are identical in every respect except the population growth rates, the high-(low-)population growth area will become labor-(capital-)abundant over time, and have a comparative advantage in the production of labor-(capital-)intensive commodity, just as predicted by the static HO model. These differences in comparative advantages must be caused by differences in autarky relative factor prices that arise due to the differences in the speed of population growth. The next corollary shows the validity of this statement.

Corollary 3

The steady state value of the wage rate, w_s, decreases in the population growth rate n, whereas that of the rental rate, r_s, increases in the population growth rate n.

The effect of the population growth rate on the steady state wage rate, w_s, depends on the sign of

$$\frac{\partial w_s}{\partial n} = \alpha\varepsilon^{\alpha}\Phi^{\frac{\alpha}{1-\alpha}-1}\frac{\partial \Phi}{\partial n}. \tag{35}$$

Since $\frac{\partial \Phi}{\partial n} < 0$, $\frac{\partial w_s}{\partial n} < 0$.

Thus, the low population growth N would have a higher wage rate than the high population growth S, explaining why the latter would have a comparative disadvantage in the production of labor-intensive commodities.[4]

The effect of the population growth rate on the steady state rental rate, r_s, can be observed through the effect of the population growth rate n on the steady state rental rate:

$$\frac{\partial r_s}{\partial n} = \alpha\varepsilon^{\alpha-1}\frac{\partial}{\partial n}\left(\frac{1}{\Phi}\right) \tag{36}$$

which is always positive, since $\frac{\partial}{\partial n}\left(\frac{1}{\Phi}\right) > 0$.

[4] This also implies that unequal population growth rates could induce labor migration from high- to low-population growth nations in the absence of barriers to labor mobility as discussed by Sayan (2003).

This implies that the region with a slowly growing population tends to have a lower rental rate on capital as compared to regions with a rapidly growing population. This is what creates the basis for its comparative advantage in the production of capital-intensive commodities and, in the absence of restrictions to capital mobility, would encourage flows of capital from capital-abundant countries to labor-abundant countries.[5]

The discussion so far unambiguously establishes that as long as $n^S > n^N$, all three of the following inequalities will hold

$$p_s^S < p_s^N, \qquad w_s^S < w_s^N, \qquad r_s^S > r_s^N \tag{37}$$

under autarky.

4. Welfare consequences of trade

Now, when two countries begin trading with each other to make use of these differences in autarky relative prices, they will specialize in the production of the commodity that they produce more efficiently, i.e., at a lower cost. This will lead to the establishment of common relative commodity and factor prices lying between respective autarky prices as predicted by the HO model and as shown by Sayan (2005). Thus, the opening of trade will lead to an increase in p_s^S and w_s^S and a decrease in r_s^S, while causing the opposite effects in the North. In other words, common values of commodity and factor prices to be established with the opening of trade will be given by the middle terms of the inequalities in (38):

$$\begin{aligned} p_s^S &< p^* < p_s^N \\ w_s^S &< w^* < w_s^N \\ r_s^S &> r^* > r_s^N \end{aligned} \tag{38}$$

The question now is whether the trade-induced changes in these autarky prices will unambiguously improve welfare for both parties as suggested by the static HO model. The following corollary answers this question.

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[5] Furthermore, capital flows induced by population aging in one region of the world can transmit the growth effects of aging globally, see Tosun (2003) and Kenç and Sayan (2001)

Corollary 4

The direction in which the steady state value of the life time utility of a citizen in *i* will change with the opening of trade is uncertain.

Proof

Obviously, the direction of change in the steady state value of the life time utility of a citizen in *i* with the switch from autarky to trade depends on the sign of the elasticity of

$$u_s{}^i = \left(c_{1ys}^{i}{}^{\theta} c_{2ys}^{i}{}^{1-\theta}\right)^{\mu} \left(c_{1os}^{i}{}^{\theta} c_{2os}^{i}{}^{1-\theta}\right)^{1-\mu}$$

with respect to the trade-induced changes in relative commodity or factor prices.

Let's consider the reaction of steady state autarky value of life time utility of an individual in country *i* to a trade-induced change in the rental for capital.[6] Given $u_s{}^i$ above, the relevant elasticity can be expressed as

$$e_{u_s{}^i, r_s{}^i} = \mu\theta e_{c_{1ys}^i, r_s{}^i} + \mu(1-\theta) e_{c_{2ys}^i, r_s{}^i} + (1-\mu)\theta e_{c_{1os}^i, r_s{}^i} + (1-\mu)(1-\theta) e_{c_{2os}^i, r_s{}^i} \quad (39)$$

where each *e* term with a subscript shows the elasticity of the steady state value of the variable denoted by the first term in the subscript with respect to the steady state value of the rental for capital, $r_s{}^i$.

Now, using (27) and (28)-(31) to obtain the steady state values of per capita consumptions in terms of $r_s{}^i$ and evaluating their elasticities with respect to $r_s{}^i$, we get

$$e_{c_{1ys}^i, r_s{}^i} = \frac{\alpha}{\alpha - 1} \quad (40)$$

$$e_{c_{2ys}^i, r_s{}^i} = \frac{\beta}{\alpha - 1} \quad (41)$$

$$e_{c_{1os}^i, r_s{}^i} = \frac{\alpha}{\alpha - 1} + \frac{r_s{}^i}{1 + r_s{}^i} \quad (42)$$

6 On account of the relative simplicity of expressions, we consider the trade-induced change in $r_s{}^i$ only. The same result will hold for $p_s{}^i$ and $w_s{}^i$ as well but the algebra to show it will be more tedious.

$$e_{c_{2os}^i, r_s^i} = \frac{\beta}{\alpha - 1} + \frac{r_s^{\ i}}{1 + r_s^{\ i}} \tag{43}$$

Substituting (40) through (43) in (39) we get

$$e_{u_s^{\ i}, r_s^{\ i}} = \mu\theta\left(\frac{\alpha}{\alpha-1}\right) + \mu(1-\theta)\left(\frac{\beta}{\alpha-1}\right) + (1-\mu)\theta\left(\frac{\alpha}{\alpha-1} + \frac{r_s^i}{1+r_s^i}\right) + (1-\mu)(1-\theta)\left(\frac{\beta}{\alpha-1} + \frac{r_s^i}{1+r_s^i}\right)$$

$$= \theta\left(\frac{\alpha}{\alpha-1}\right) + (1-\theta)\left(\frac{\beta}{\alpha-1}\right) + (1-\mu)\left(\frac{r_s^{\ i}}{1+r_s^{\ i}}\right) \tag{44}$$

But

$$e_{u_s^{\ i}, r_s^{\ i}} \begin{cases} < 0 & if \ \dfrac{1}{r_s^{\ i}} = \dfrac{(1-\mu)(1-\alpha)}{\theta\alpha + (1-\theta)\beta} - 1 \\ > 0 & otherwise \end{cases} \tag{45}$$

implying that the sign of the elasticity depends on the initial values of parameters.

The autarky steady state values of $r_s^{\ i}$ will change in opposite directions in N and S after opening of trade as shown in inequality (38). Since the elasticity in (44) is ambiguous in sign, however, the direction of trade-induced change in $u_s^{\ i}$ will not be as straightforward to tell as in the static HO model. This result is consistent with previously cited OLG-GE studies based on stationary populations, as well as the recent work by Sayan (2005) who considers a dynamic OLG-GE extension of the HO model with population growth differentials.

5. Conclusions

Our discussion of the closed-form solutions to the 2x2 OLG model in the paper has shown concerning two countries that are identical in every respect except the population growth rates that the high-(low-)population growth country will become labor-(capital-) abundant over time, and must be expected to have a comparative advantage in the production of labor-(capital-)intensive commodity, as suggested by the static HO model which assumes away differences in technologies and preferences to focus on the effects of disparities in relative factor endowments.

Our analysis has indeed revealed that when the model is solved under autarky, differences in the population growth rates alone are capable of giving rise to comparative advantages by creating differences in relative factor endowments and hence leading to different relative prices across countries, regardless of initial population sizes of trading countries. In other words, the only difference in demographic characteristics that matters for the direction of product and factor flows is the one between population growth rates. Thus, an examination of the sensitivity of the steady state value of relative price ratio under autarky to changes in population growth rate will identify directions of comparative advantages correctly and consistently with the static HO framework.

The welfare effects of trade between two countries, however, were found to depend on the values of system parameters, and hence, were ambiguous. This is a significant finding adding inequality of population growth rates to the previously reported reasons explaining why welfare results of the standard HO model might not hold under different dynamic set-ups.

Aside from its theoretical significance, the analysis here provides the grounds for further discussion of the likely consequences ahead of the differential speed of demographic transition and population aging between developing and developed areas in the world today. Even though the employment of standard HO assumptions that countries have identical production technologies and preferences may be deemed unrealistic and appear to limit the relevance of results to real life, the analysis here successfully underlines the role that the observed differences in the speed of population growth is likely to play as a major determinant of trade and factor flows between developing and developed countries in the following decades. Furthermore, given the increasing ease of technology transfers and converging tastes of consumers across nations due to the accelerating pace of globalization, the framework here may not be that unrealistic.

Currently, developed countries (or the North) cope with population aging and the resulting decline in the share of working population by employing more capital per worker than in the South so as to keep labor productivity higher. Yet, this will not necessarily prevent the high population growth countries of the South from dominating the world output and trade through larger volumes of output and exports in the future. Even now, China provides an example to this potential of developing countries. Thus, additional research is certainly required to explore the paths that relative commodity and factor prices will follow into the long-run as relative factor endowments continuously change due to differential speeds of fertility decline and population aging in the South and the North. An interesting question within this context would be to investigate the effects of labor saving technologies that the North could potentially introduce on a continuous basis.

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Özet

Nüfus artış hızı farklarının uluslararararası ticarete etkileri: Heckscher-Ohlin çerçevesinde bir çakışan kuşaklar-genel denge analizi

Bu makale nüfusları farklı hızlarda artan iki ülkeli bir dünyayı ele almakta ve nüfus dinamiklerinin uluslararası ticaret üzerindeki etkilerini, iki-sektörlü ve iki-faktörlü bir çakışan kuşaklar-genel denge modelinden elde edilen analitik çözümlerden çıkan sonuçlar ışığında incelemektedir. Statik Heckscher-Ohlin (HO) modelinde olduğu gibi, ticaret yapacak tarafların tüketici tercihleri ve üretim teknolojilerinin birbirinin aynı olduğu varsayımından yola çıkılarak elde ettiğimiz sonuçlar, nüfus artış hızı farklarının, statik HO modelinde öngörülene benzer biçimde kıyaslamalı üstünlüklerin ortaya çıkmasına yol açacağını göstermiştir. Ancak HO modelinin öngördüğünden farklı olarak, böyle bir dinamik çerçevede serbest ticaretin her iki taraf için refah kazanımı sağlamayabileceği ispatlanmıştır.