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Central bank independence: Optimal conservativeness, and optimal term lengths *

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Abstract

In many economies, the legal term length of office of central banker is specified by a constitutional rule set by the Government. However, this rule is effective as long as the central banker pursues credibly independent policies, and as long as the Government commits to the rule. On the other hand, the policies of the central banker must also be sufficiently flexible to respond to output shocks. In this paper, we determine that the Government can obtain the balance between credibility and flexibility by appointing an optimally weight-conservative central banker who implements time-consistent policies for an optimal period of time.

1. Introduction

One of the most compelling questions in the design of monetary policy is, how much flexibility, or independence, should be granted to the monetary authority. It is a generally held view that whenever the market determined output or employment level is suboptimal, a monetary shock to stimulate output to overcome short-run rigidities at the cost of some inflation is appropriate. Anticipating this incentive, wage-setters would set a sufficiently high nominal wage inflation in their wage contracts in an attempt to discourage the policymaker from reducing the real wage by engaging in surprise inflation. Even though the policymaker

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is perfectly benevolent, a fully discretionary time-consistent policy would result in an excessively high inflation, or an inflationary bias, without any gains in terms of output and employment. According to the seminal paper by Kydland and Prescott (1977), attempts to eliminate the inflationary bias through committing to a credible, simple rules policy would prove to be optimal, however dynamically inconsistent.

In addressing Kydland and Prescott, Rogoff (1985) shows that delegating the conduct of monetary policy to an independent central banker that places a lower weight on output stabilization and a higher weight on inflation stabilization relative to the policymaker helps ameliorate the dynamic inconsistency problem. As per Rogoff, appointing a central banker who is relatively more weight-conservative than the policymaker in terms of inflation stabilization reduces the time-consistent rate of inflation, albeit at the expense of a distorted response to output shocks, especially when output shocks are extreme.

Following Rogoff's proposition, Lohmann (1992) introduces the optimal design of a central banking institution which credibly follows a low-inflation monetary policy as it responds to extreme output shocks by implementing a flexible escape clause. In Lohmann's proposed institutional design, the appointed weight-conservative central banker knows that she will be overridden by the policymaker when output shocks are extreme, and accommodates the policymaker's ex-post demands in order to avoid being overridden. In 'normal' times however, the central banker conducts a low-inflation policy independently at her own discretion. According to Lohmann, the proposed instutional design dominates other central banking institutional designs, such as the fully discretionary regime, the regimes of full or partial commitment to the simple zero-inflation rule, and the institution of a fully independent, weight-conservative central banker.

In contrast to Rogoff, Alesina and Summers (1993) find empirical evidence that while central bank independence reduces the level and variability of inflation, it has no clear impact on output variability, at least in developed countries. This finding suggests that an independent central banker does not necessarily compromise flexibility for credibility in response to output shocks. Waller and Walsh (1996) posit a framework that accounts for these empirical findings. They parameterize central bank independence in terms of central banker's legal term length of office and political partisanship, while still staying in the realm of Rogoff's weight-conservative central banker. Waller and Walsh argue that appointing a weight-conservative independent central banker lengthens the optimal term of office while increasing the variability of output. A long term of office, on the other hand, reduces partisan influences and election-induced output variability. As a result, the two opposing effects cancel each other out. However in Waller and Walsh, the degree of weight-conservativeness of the central banker (‘type of the central banker’) is given exogenously, hence we cannot determine whether the appointed central banker is the optimal choice given existing conditions in the economy. Furthermore, the direction of causality between the type of the central banker and the optimal term length is not convincing.

In this paper, as in Waller and Walsh, in addition to weight-conservativeness, a ‘constitutional rule’ in the form of an optimal term length of office describes central bank independence. Ex-ante, the Government determines the optimal term length of office of central banker by maximizing the infinite horizon social welfare. The Government is presumed to obey this rule. An optimal type central banker à la Rogoff is then appointed to conduct monetary policy during that term length. By committing to the constitutional rule and by appointing an optimal type central banker, the Government can successfully balance the trade-off between credibility and flexibility, given the severity of output shocks and the existing conditions of the economy.

In addition to Waller and Walsh, other theoretical studies on legal term lengths include O’Flaherty (1990), Garfinkel and Oh (1993), and Lin (1999) as a Comment to Waller and Walsh. As for the empirical studies, Cukierman, Webb, and Neyapti (1992), and Cukierman (1992) take the actual central bank governor turnover rates as a proxy for the legal term length, i.e., they do not examine the legal term lengths, per se. Nevertheless, they find that in developing countries, the rate of turnover for the governors contribute significantly in explaining inflation, and it is even more significant in explaining variations in inflation across sampled developing countries.

O’Flaherty (1990) analyzes the optimal term length within a dynamic principal-agent framework of the public and the central bank. According to O’Flaherty, one-period terms are not optimal because the central bank always inflates in the first period of office. Infinite terms of office are also not desirable because otherwise the public would be permitting the central banker to ‘run wild’, in other words, the public needs to be able to dismiss any central banker who does not implement the public’s desired policy. This finding is in accord with Jean Tirole's (Tirole, 1994) argument that short-terms serve as a check against bad policy decisions. Thus, in O’Flaherty’s work, optimal term length is always finite.

Garfinkel and Oh (1993), do not analyze optimal term length per se, but describe a multi-period monetary policy targeting procedure. They present a model of monetary policy with private information in a multi-period setting and analyze the monetary authority's optimal, time-consistent monetary policy subject to the N-period average targeting constraint. Under this procedure, the longer the targeting horizon, the greater is the degree of flexibility permitted in policy. However, as the targeting horizon increases, the greater is the monetary authority's willingness to act upon its incentive to create surprise inflation, thus the greater is the credibility problem. Hence, the optimal targeting horizon provides the optimal trade-off between flexibility and credibility. They further suggest that as the monetary authority attaches more importance to its objective for inflation relative to its objective for output, the optimal targeting horizon becomes longer. However, as in Waller and Walsh, in their study the type of the central banker is exogenously given. In this paper, we determine the type of the central banker endogenously, given the conditions in the economy (so as to assure time-consistent policies) and also the optimal term length of the central banker; hence, the choice of central banker is not arbitrary. Furthermore, we show that a positive relationship exists between optimal term length and the optimal weight the central banker puts on inflation stabilization. As in Lin (1999), this result suggests a strategic complementarity between the optimal term length and the optimal weight conservativeness.

Lin, in a more simplified model, extends the theoretical framework in Waller and Walsh to endogenize the optimal type of central banker. According to Lin's results, the optimal term length of the optimal type central banker is finite, even if the optimal central banker is conservative enough in terms of inflation stabilization. The present study introduces a simpler environment than the one in Lin, and we establish that the term length of office of the optimal type central banker may be finite or infinite, depending on the probability that the appointed central banker assumes a different (non-optimal) type during her term of office in the periods following her period of appointment.

The rest of the paper is organized as follows: in Section 2, we introduce the basic model. In Section 3, we define equilibria for three different cases, first for the case where the Government delegates the conduct of policy to a central banker for single-period terms; second, for the case where delegation occurs for multi-period terms, and the central banker's type stays the same during the term; finally, for the case where the central banker shirks with some small probability during the term. In Section 4, we proceed to determine the optimal term length of delegation of policy in the last two cases. Section 5 includes a discussion on comparative statics. In Section 6, we conclude.

2. The basic model

We consider an economy characterized by the following supply function:

$$y_t = \pi_t - \omega_t + \varepsilon_t \tag{1}$$

where y_t is the log of output, π_t is the inflation rate, ω_t is the nominal wage rate determined by wage-setters at time t. The output shock ε_t is i.i.d. and is assumed to be normally distributed with zero mean and finite variance equal to σ_ε^2. The nominal wages ω_t are based on rational expectations concerning inflation π_t:

$$\omega_t = E_\varepsilon \pi_t \tag{2}$$

where E_ε is the expectations operator. Inflationary expectations are denoted by π_t^e.

The Government has the same preferences as society, and maximizes the per period utility given as

$$U_{GOV} = -\frac{A}{2}(y_t - \bar{y})^2 - \frac{\pi_t^2}{2}$$

(3)

at time t, where $\bar{y} > 0$ is the society's desired level of (log) output, and $A > 0$ denotes the Government's type, i.e., A is the relative weight that the Government puts on output stabilization versus inflation stabilization. For simplicity, we assume that the Government is infinitely-lived, i.e., there are no elections that may change the Government's type over time.

Two alternative regimes are described to complete the model. In the case of 'full discretion', inflation π_t is the Government's policy variable, and π_t is determined after inflationary expectations are set and output shock ε_t is observed. The Government solves the utility maximization problem

$$\max_{\pi_t} U_{GOV}$$

$$\text{subject to } y_t = \pi_t - \pi_t^e + \varepsilon_t \tag{4}$$

in every period. Subscript t is dropped since the problem is identical in each period. From the first order conditions for this problem, we find that the Government maximizes utility at

$$\pi(\pi^e, \varepsilon) = \frac{A}{1+A}(\pi^e - \varepsilon + \bar{y}) \tag{5}$$

Under rational expectations,

$$\begin{aligned}\pi^e &= E_\varepsilon[\pi(\pi^e,\varepsilon)] \\ &= E_\varepsilon\left[\frac{A}{1+A}(\pi^e-\varepsilon+\bar{y})\right] \\ &= A\bar{y}\end{aligned} \tag{6}$$

The inflationary bias $A\bar{y}$ in (6) is substituted into (5) to obtain inflation as

$$\pi(\varepsilon) = A\bar{y} - \frac{A}{1+A}\varepsilon \tag{7}$$

and the corresponding (log) output as

$$y(\varepsilon) = \frac{1}{1-A}\varepsilon\,. \tag{8}$$

In the case of 'delegation', Rogoff (1985) shows that delegation of policy to a conservative central banker, with a relatively lower weight on output stabilization than the Government, will achieve lower inflationary bias. In what follows, we determine whether the optimal central banker chosen to implement the policy is in fact relatively more conservative than the Government, or not.

Consider the following sequence of events in a single period of time[1]: (i) π^e is set by wage-setters; (ii) central banker of type λ is chosen by the Government; (iii) output shock ε is realized; and finally, (iv) policy π is chosen by central banker who has preferences described by the utility function

$$U_{CB} = -\frac{\lambda}{2}(y-\bar{y})^2 - \frac{\pi^2}{2} \tag{9}$$

where λ specifies central banker's type.

Next, delegation of policy is studied in three versions of the basic model, labeled Model 1, Model 2, and Model 3, respectively.

3. Delegation and optimal conservativeness

Let T denote the term length of office of central banker to be appointed. Three cases are considered:

[1] See Lossani *et al*. (1998) for the treatment of the same sequence of events as ours, in a two-period model.

3.1 Single period terms (model 1)

In the first case, the term of office of the central banker is assumed to be one period, $T = 1$, i.e., the Government appoints a new central banker every period.

Definition 1. An equilibrium for Model 1 economy with a single period term length is a list $(\hat{\lambda}, \hat{\pi}^e, \hat{\pi}, \hat{y})$ such that for all $A, \sigma_\varepsilon^2, \bar{y}$,

i) given π^e and ε, central banker of type $\hat{\lambda}$ chooses $\pi(\hat{\lambda}, \hat{\pi}^e, \varepsilon)$ to solve

$$\max_{\pi} -\frac{\hat{\lambda}}{2}(y-\bar{y})^2 - \frac{\pi^2}{2}$$

$$\text{subject to } y = \pi - \pi^e + \varepsilon \tag{10}$$

ii) given π^e, $\pi(\hat{\lambda}, \pi^e, \varepsilon)$ and $y(\hat{\lambda}, \pi^e, \varepsilon)$, $\hat{\lambda}$ maximizes the Government's expected utility

$$E_\varepsilon(U_{GOV}) = E_\varepsilon\left[-\frac{A}{2}(y(\hat{\lambda}, \pi^e, \varepsilon) - \bar{y})^2 - \frac{1}{2}\pi(\hat{\lambda}, \pi^e, \varepsilon)^2\right]$$

$$\text{subject to } y(\hat{\lambda}, \pi^e, \varepsilon) = \pi(\hat{\lambda}, \pi^e, \varepsilon) - \pi^e + \varepsilon \tag{11}$$

iii) $\hat{\pi}^e = E_\varepsilon\left[\pi(\hat{\lambda}, \hat{\pi}^e, \varepsilon)\right]$

iv) $\hat{\pi}(\varepsilon) = \pi(\hat{\lambda}, \hat{\pi}^e, \varepsilon)$

v) $\hat{y}(\varepsilon) = y(\hat{\lambda}, \hat{\pi}^e, \varepsilon)$

Claim 1. In an equilibrium for Model 1, $\hat{\lambda} = A$, then the inflationary bias is $\pi^e = A\,\bar{y}$.

Proof. We solve for the equilibrium backwards, starting with the central banker's problem.

Step 1. Given π^e, any central banker of type λ solves the maximization problem

$$\max_{\pi} -\frac{\lambda}{2}(y-\bar{y})^2 - \frac{\pi^2}{2}$$

subject to $y = \pi - \pi^e + \varepsilon$

From the first order conditions of this problem, we have

$$\pi(\lambda, \pi^e, \varepsilon) = \frac{1}{1+\lambda}(\pi^e + \varepsilon + \bar{y}) \tag{12}$$

The corresponding (log) output is

$$y(\lambda, \pi^e, \varepsilon) = \frac{1}{1+\lambda}(-\pi^e + \varepsilon + \lambda \bar{y}) \quad (13)$$

Note that expectations over $\pi(\lambda, \pi^e, \varepsilon)$ cannot be taken at this stage, since the wage-setters cannot yet observe the central banker of type λ at the time they set their inflationary expectations.

Step 2. Note that the output shock ε is realized after central banker is chosen, that is, the Government acts to choose the central banker before the output shock is realized; hence expectations over ε are taken. The Government maximizes expected utility by appointing the central banker who chooses policy $\pi(\lambda, \pi^e, \varepsilon)$ and (log) output $y(\lambda, \pi^e, \varepsilon)$. The expected utility of the Government is given by

$$E_\varepsilon(U_{GOV}) = E_\varepsilon\left[-\frac{A}{2}\left(y(\lambda,\pi^e,\varepsilon)-\bar{y}\right)^2 - \frac{\pi(\lambda,\pi^e,\varepsilon)^2}{2}\right]$$

$$= \left[\frac{-A-\lambda^2}{2(1+\lambda)^2}\right]\left(\sigma_\varepsilon^2 + (\pi^e+\bar{y})^2\right) \quad (14)$$

The central banker type λ maximizing the Government's expected utility in (14) is given by

$$\left[\frac{A-\lambda}{(1+\lambda)^3}\right]\left[\sigma_\varepsilon^2 + (\pi^e+\bar{y})^2\right] = 0 \quad (15)$$

from which it follows that

$$\hat{\lambda} = A \quad (16)$$

In other words, the Government chooses a central banker whose policy position is exactly the same as the Government's. After substituting for the optimal type of central banker in the inflation (12) chosen by the central banker, we obtain

$$\hat{\pi}(\pi^e,\varepsilon) = \frac{A}{1+A}(\pi^e - \varepsilon + \bar{y}) \quad (17)$$

Taking expectations on both sides and rearranging, we get the inflationary bias as

$$\pi^e = A\,\bar{y} \quad (18)$$

Q.E.D.

Finally, substituting (16) and (18) in (12) and (13), the equilibrium inflation and (log) output are obtained as

$$\hat{\pi}(\varepsilon) = A\bar{y} - \frac{A}{1-A}\varepsilon \tag{19}$$

$$\hat{y}(\varepsilon) = \frac{1}{1+A}\varepsilon \tag{20}$$

Therefore, if the central banker is appointed for a single-period term, we conclude that in terms of inflationary bias, there is no gain in delegating policy to an independent central banker. Intuitively, based on the "implicit" information given in the term length, wage-setters anticipate that the government will choose a central banker who has the same type as itself, and thus they set their inflationary expectations taking this information into consideration when they bargain for wages. This outcome is in accordance with O'Flaherty's result concerning one-period-term appointments.

In Models 2 and 3, in a multi-period term environment, it is established that the Government optimally delegates the implementation of policy to a central banker who is relatively more conservative in terms of inflation stabilization. At the beginning of time, the Government sets the term length. The wage-setters observe this term length when they set their inflationary expectations, π_0^e in the first period. But the wage-setters cannot yet observe the type of central banker appointed in the first period when they set π_0^e. In subsequent periods, however, they learn the type of the central banker. They adjust their inflationary expectations in periods following the first period, taking the information about the type of the central banker implementing the policy as given.

In Model 2, it is assumed that the central banker's type remains the same throughout her term of office. In Model 3, with a small probability the central banker's type may switch in the periods following the appointment period, i.e., the central banker shirks.

3.2. Multi-period terms, fixed type (model 2)

Definition 2. An equilibrium for the Model 2 economy with a given term length of office of central banker, *T*, is a sequence of inflation rates $\{\hat{\pi}_t(\varepsilon_t)\}_{t=0}^{T-1}$, (log)output levels $\{\hat{y}_t(\varepsilon_t)\}_{t=0}^{T-1}$ and a type λ which depends on $A, T, \sigma_\varepsilon^2, \bar{y}$ and a discount factor $0<\beta<1$, such that for all $A, T, \sigma_\varepsilon^2, \bar{y}, \beta$,

i) given π_0^e and ε_t for all *t*, central banker of type $\hat{\lambda}$ chooses $\{\pi_t(\hat{\lambda}, \pi_0^e, \varepsilon_t)\}_{t=0}^{T-1}$ to solve

$$\max_{\pi_t} -\frac{\hat{\lambda}}{2}(y_t - \bar{y})^2 - \frac{\pi_t^2}{2}$$

subject to $y_t = \pi_t - \pi_t^e + \varepsilon_t$, where $\pi_t^e = E_{\varepsilon_t}\pi_t$ (21)

ii) given π_0^e, $\{\pi_t(\hat{\lambda}, \pi_0^e, \varepsilon_t)\}_{t=0}^{T-1}$ and $\{y_t(\hat{\lambda}, \pi_0^e, \varepsilon_t)\}_{t=0}^{T-1}$, $\hat{\lambda}$ maximizes the Government's expected discounted utility

$$E_\varepsilon(U_{GOV}) = \sum_{t=0}^{T-1}\beta^t E_{\varepsilon_t}\left[-\frac{A}{2}\left(y_t(\hat{\lambda}, \pi_0^e, \varepsilon_t) - \bar{y}\right)^2 - \frac{\pi_t(\hat{\lambda}, \pi_0^e, \varepsilon_t)^2}{2}\right] \quad (22)$$

subject to $y_t(\hat{\lambda}, \pi_0^e, \varepsilon_t) = \pi_t(\hat{\lambda}, \pi_0^e, \varepsilon_t) - \pi_t^e + \varepsilon_t$

iii) $\hat{\pi}_0^e = E_\varepsilon[\pi_0(\hat{\lambda}, \hat{\pi}_0^e, \varepsilon_t)]$

iv) $\hat{\pi}_t(\varepsilon_t) = \pi_t(\hat{\lambda}, \hat{\pi}_0^e, \varepsilon_t)$

v) $\hat{y}_t(\varepsilon_t) = y_t(\hat{\lambda}, \hat{\pi}_0^e, \varepsilon_t)$

Claim 2. In an equilibrium for Model 2 with multi-period term appointments, given the initial inflationary expectations π_0^e, there exists a unique optimal central banker type $\hat{\lambda}$ such that $0 < \hat{\lambda} < A$.

Proof. (Existence) We solve for the equilibrium backwards, starting with the central banker's problem:

Step 1. Given π_0^e, any central banker of type λ solves

$$\max_{\pi_t} -\frac{\lambda}{2}(y_t - \bar{y})^2 - \frac{\pi_t^2}{2}$$

subject to $y_t = \pi_t - \pi_t^e + \varepsilon_t$, where $\pi_t^e = E_{\varepsilon_t}\pi_t$ (23)

From the first order condition, for all *t,* inflation is obtained as

$$\pi_t(\lambda, \pi_t^e, \varepsilon_t) = \frac{\lambda}{1+\lambda}(\pi_t^e - \varepsilon_t + \bar{y}) \quad (24)$$

and the corresponding (log) output as

$$y_t(\lambda, \pi_t^e, \varepsilon_t) = \frac{\lambda}{1+\lambda}(-\pi_t^e + \varepsilon_t + \lambda\bar{y}) \quad (25)$$

In $t = 0$,

$$\pi_0(\lambda, \pi_0^e, \varepsilon_0) = \frac{\lambda}{1+\lambda}(\pi_0^e - \varepsilon_0 + \bar{y}) \quad (26)$$

$$y_0(\lambda,\pi_0^e,\varepsilon_0)=\frac{\lambda}{1+\lambda}(-\pi_0^e+\varepsilon_0+\lambda\bar{y}) \tag{27}$$

Notice that the choice of inflation and the corresponding (log) output in the first period of a multi-period term model are the same as those in the single-period term model. Also note that in t = 0, one cannot take expectations over $\pi_0(\lambda,\pi_0^e,\varepsilon_0)$ since λ is not yet observed by the wage-setters. However, in all subsequent periods, the wage-setters observe the appointed central banker's type. Having observed the central banker's type, at time t = 1, 2,…, T-1,

$$\pi_t^e = E_{\varepsilon_t}[\pi_t(\lambda,\pi_t^e,\varepsilon_t)] = E_{\varepsilon_t}\left[\frac{\lambda}{1+\lambda}(\pi_t^e-\varepsilon_t+\bar{y})\right] \tag{28}$$

Rearranging yields

$$\pi_t^e = \lambda\bar{y} \tag{29}$$

Substituting (29) back in (24) and (25) yields

$$\pi_t(\lambda,\varepsilon_t)=\lambda\bar{y}-\frac{\lambda}{1+\lambda}\varepsilon_t \tag{30}$$

and

$$y_t(\lambda,\varepsilon_t)=\frac{\lambda}{1+\lambda}\varepsilon_t \tag{31}$$

Step 2. The Government's choice of optimal central banker problem is

$$\overbrace{\max_{\lambda}\sum_{t=0}^{T-1}\beta^t E_{\varepsilon_t}\left[-\frac{A}{2}(y_t-\bar{y})^2-\frac{\pi_t^2}{2}\right]}^{E_\varepsilon(U_{GOV})} \tag{32}$$

subject to (26), (27), (30) and (31).

After substituting (26) for π_0, (27) for y_0, (30) for π_t and (31) for y_t for all t = 1, 2, …, T-1 in (32), the Government's problem can be rewritten as

$$\max_{\lambda}\left[\frac{-A-\lambda^2}{2(1+\lambda)^2}\right]\left[\left(\frac{1-\beta^T}{1-\beta}\right)\sigma_\varepsilon^2+(\pi_0^2+\bar{y})^2\right]+\beta\left(\frac{1-\beta^{T-1}}{1-\beta}\right)\bar{y}^2\left(\frac{-A-\lambda^2}{2}\right) \tag{33}$$

The first order condition for (33) is

$$\frac{\partial E_\varepsilon(U_{GOV})}{\partial\lambda}=\left[\frac{A-\lambda}{(1+\lambda)^3}\right]\left[\left(\frac{1-\beta^T}{1-\beta}\right)\sigma_\varepsilon^2+(\pi_0^2+\bar{y})^2\right]-\beta\left(\frac{1-\beta^{T-1}}{1-\beta}\right)\bar{y}^2\lambda=0 \tag{34}$$

Note that if $\hat{\lambda} = 0$ then $\frac{\partial E_\varepsilon(U_{GOV})}{\partial\lambda} > 0$, and if $\hat{\lambda} = A$, $\frac{\partial E_\varepsilon(U_{GOV})}{\partial\lambda} < 0$, then, by the Mean Value Theorem, there exists some $\hat{\lambda}$ such that $0 < \hat{\lambda} < A$ which satisfies $\frac{\partial E_\varepsilon(U_{GOV})}{\partial\lambda} = 0$.

(Uniqueness) To show the uniqueness of the optimal type central banker, we rewrite the first order condition (34) as:

$$\lambda = \frac{A\Gamma}{\Gamma + \Phi(1+\lambda)^3} \tag{35}$$

The function *F(λ)* is then defined as follows:

$$F(\lambda) \equiv \frac{A\Gamma}{\Gamma + \Phi(1+\lambda)^3} \tag{36}$$

for all λ where

$$\Gamma \equiv \left(\frac{1-\beta^T}{1-\beta}\right)\sigma_\varepsilon^2 + (\pi_0^e + \bar{y})^2$$

and

$$\Phi \equiv \beta\left(\frac{1-\beta^{T-1}}{1-\beta}\right)\bar{y}^2$$

The function *F(λ)* is monotonically decreasing in λ as

$$F(0) = \frac{A\Gamma}{\Gamma+\Phi} > 0$$

$$\frac{\partial F(\lambda)}{\partial\lambda} = -\frac{A\Gamma\Phi 3(1+\lambda)^2}{\left[\Gamma + \Phi(1+\lambda)^3\right]^2} < 0$$

and

$$\lim_{\lambda\to\infty} F(\lambda) = 0$$

Thus,

$$0 < F(\lambda) < \frac{A\Gamma}{\Gamma+\Phi}, \forall\lambda > 0$$

Next, note that the left-hand-side of (35) is a straight 45-degree line through the origin. Since $F(\lambda)>0$ and $\frac{\partial F(\lambda)}{\partial\lambda} < 0$, $F(\lambda)$ intersects the 45-degree line at one and only one point, say $\hat{\lambda}$. Moreover, since

$0 < F(\lambda) < \dfrac{A\Gamma}{\Gamma+\Phi} < A,\ \forall\lambda > 0$, the intersection occurs at a value of λ which is bounded between 0 and $\dfrac{A\Gamma}{\Gamma+\Phi}$. Q.E.D.

We observe that the solution to the Government's choice of optimal central banker problem depends on an exogenously given initial value of inflationary expectations. In an equilibrium, however, initial inflationary expectations must be set by wage-setters to satisfy

$$\pi_0^e = \hat{\lambda}(\pi_0^e)\bar{y} \tag{37}$$

In the following Claim, it is shown that such initial inflationary expectations value is in fact unique:

Claim 3. There exists a unique value of equilibrium inflationary expectations $\hat{\pi}_0^e$ such that

$$\hat{\pi}_0^e = \hat{\lambda}(\hat{\pi}_0^e)\bar{y}$$

Proof. We rewrite (37) as

$$\pi_0^e = H(\pi_0^e) \tag{38}$$

where $H(\pi_0^e)$ is defined as

$$H(\pi_0^e) \equiv \hat{\lambda}(\pi_0^e)\bar{y} = \frac{A\Gamma\bar{y}}{\Gamma+\Phi(1+\hat{\lambda})^3} \tag{39}$$

in which Γ and Φ are defined as above.

The function $H(\pi_0^e)$ is monotonically increasing in π_0^e as

$$H(0) > 0,$$

$$\frac{\partial H(\pi_0^e)}{\partial \pi_0^e} = \frac{2(\pi_0^e+\bar{y})\Gamma\left[1+\Phi(1+\hat{\lambda})^3 - A\right]}{\left[\Gamma+\Phi(1+\hat{\lambda})^3\right]^2} > 0,$$

$$\lim_{\pi_0^e \to A\bar{y}} H(\pi_0^e) < A\bar{y}$$

Note that the left hand-side of (39) is a straight 45^0-line through the origin. Since $H(0) > 0$, $\dfrac{\partial H(\pi_0^e)}{\partial \pi_0^e} > 0$ and $H(\pi_0^e)$ is bounded above by $A\,\bar{y}$, the function H intersects the 45-degree line at exactly one point, say $\hat{\pi}_0^e$, which is also bounded above by $A\,\bar{y}$. Q.E.D.

*Theorem 1.*Model 2 has a unique equilibrium.

Proof. The result follows from Claims 2 and 3.

Finally, one can conclude that in a multi-period term economy, the inflationary bias is lower than that of in a single-period term economy.

3.3 Multi-period terms, central banker shirks (model 3)

In this section, we consider the probability that the incumbent central banker's type may switch in the periods following the first period of her term of office. Every period, with a small probability 1-μ, 0<μ≤1, the central banker's type switches to some $\overline{\lambda} > 0, \overline{\lambda} > \hat{\lambda}$. Without loss of generality, the case where $\overline{\lambda} = A$ is considered. If the central banker's type switches to *A* at any period during her term, she stays as type *A* in the remaining periods. The probability that the central banker's type switches is common knowledge.

Definition 3. An equilibrium for the Model 3 economy with a given term length of office of central banker, *T,* is a sequence of inflation rates $\{\hat{\pi}_t(\varepsilon_t)\}_{t=0}^{T-1}$, (log) output levels $\{\hat{y}_t(\varepsilon_t)\}_{t=0}^{T-1}$, and a type $\hat{\lambda}_\mu$ which depends on *A*, β, *T*, $\sigma_\varepsilon^2, \overline{y}$ and a large μ,

i) given π_0^e and ε_t for all *t,* central banker of type $\hat{\lambda}_\mu$ chooses $\{\pi_t(\hat{\lambda}_\mu, \pi_0^e, \varepsilon_t)\}_{t=0}^{T-1}$ to solve

$$\max_{\pi_t} -\frac{\hat{\lambda}_\mu}{2}(y_t - \overline{y})^2 - \frac{\pi_t^2}{2} \tag{40}$$

subject to $y_t = \pi_t - \pi_t^e + \varepsilon_t$ and $\pi_t^e = E_{\varepsilon_t}\pi_t$

ii) given π_0^e, $\{\pi_t(\hat{\lambda}_\mu, \pi_0^e, \varepsilon_t)\}_{t=0}^{T-1}$ and $\{y_t(\hat{\lambda}_\mu, \pi_0^e, \varepsilon_t)\}_{t=0}^{T-1}$, $\hat{\lambda}_\mu$ maximizes the Government's expected discounted utility:

$$E_\varepsilon(U_{GOV}) = \sum_{t=0}^{T-1}\beta^t E_{\varepsilon_t}\left\{\mu^t\left[-\frac{A}{2}\left(y_t(\hat{\lambda}_\mu, \pi_0^e, \varepsilon_t) - \overline{y}\right)^2 - \frac{1}{2}\pi_t(\hat{\lambda}_\mu, \pi_0^e, \varepsilon_t)^2\right] + (1-\mu^t)\left[-\frac{A}{2}\left(\tilde{y}_t(\varepsilon_t) - \overline{y}\right)^2 - \frac{\tilde{\pi}_t(\varepsilon_t)^2}{2}\right]\right\} \tag{41}$$

subject to

$$y_t(\hat{\lambda}_\mu, \pi_0^e, \varepsilon_t) = \pi_t(\hat{\lambda}_\mu, \pi_0^e, \varepsilon_t) - \pi_t^e + \varepsilon_t$$

$$\tilde{y}_t(\varepsilon_t) = \frac{A}{1+A}\varepsilon_t$$

and $\tilde{\pi}_t(\varepsilon_t) = A\bar{y} - \frac{A}{1+A}\varepsilon_t$

iii) $\hat{\pi}_0^e = E_\varepsilon[\pi_0(\hat{\lambda}_\mu, \hat{\pi}_0^e, \varepsilon_t)]$

iv) $\hat{\pi}_t^e(\varepsilon_t) = \pi_t(\hat{\lambda}_\mu, \hat{\pi}_0^e, \varepsilon_t)$

v) $\hat{y}_t(\varepsilon_t) = y_t(\hat{\lambda}_\mu, \hat{\pi}_0^e, \varepsilon_t)$

*Theorem 2.*Model 3 has a unique equilibrium.

Proof. Same as Proof of Theorem 1.

Comparing the optimal central banker types[2] from Models 2 and 3, one can make the observation that for any $\mu \in (0,1]$, it must be the case that $\hat{\lambda} \leq \hat{\lambda}_\mu$. In fact, a high μ (or, a low probability to shirk) signals a higher commitment to the initial policy by the central banker, and thus signals a higher inflationary stability to the wage-setters, than with the case with a low μ.

4. Optimal term lengths

In this part of the study, it is again assumed that the Government is infinitely lived. The Government, in Models 2 and 3, chooses the optimal term length of central banker at the beginning of time. In other words, ex-ante, the Government's objective is to maximize expected discounted utility with respect to the term length of optimal type central banker.

First, the optimal term length of a central banker given in Model 2 is studied.

4.1 Optimal term length when central banker's type is fixed (special case with $\mu = 1$)

Let $\hat{\lambda}, \{\hat{\pi}_t\}_{t=0}^{T-1}, \{\hat{y}_t\}_{t=0}^{T-1}$ be an equilibrium given *T*, and let $V_T(T)$ denote the discounted value of the Government's expected utility in a given term length of *T* periods, that is,

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[2] Please see Appendix E for a derivation.

$$V_T(T) = \sum_{t=0}^{T-1} \beta^t E_{\varepsilon_t}\left[-\frac{A}{2}(\hat{y}_t - \bar{y})^2 - \frac{\hat{\pi}_t^2}{2} \right] \quad (42)$$

Recall that an infinitely lived Government is assumed, hence the Government's expected discounted utility over an infinite lifetime is defined as follows:

$$E_\varepsilon[W_{GOV}(T)] = \sum_{\tau=0}^{\infty} \beta^{\tau T} \sum_{t=0}^{T-1} \beta^t E_{\varepsilon_t}\left[-\frac{A}{2}(\hat{y}_t - \bar{y})^2 - \frac{\hat{\pi}_t^2}{2} \right] \quad (43)$$

Equivalently, one can rewrite $E_\varepsilon[W_{GOV}(T)]$ as

$$E_\varepsilon[W_{GOV}(T)] = \sum_{\tau=0}^{\infty} \beta^{\tau T} \sum_{t=0}^{T-1} \beta^t E_{\varepsilon_t}\left[-\frac{A}{2}(\hat{y}_t - \bar{y})^2 - \frac{\hat{\pi}_t^2}{2} \right] \quad (44)$$

where $V_T(T)$ is defined as in (42).

Lemma 1. Equilibrium inflationary expectations $\hat{\pi}_0^e$ are monotonically decreasing in term length *T*.

Proof. Appendix A shows that if we define $\pi_0^e = H(\pi_0^e) \equiv \hat{\lambda}(\pi_0^e)\bar{y}$, then, $\frac{\partial H(\pi_0^e)}{\partial T} < 0$, implying that when *T* increases, the value of *H* decreases for any π_0^e, hence the *H* curve shifts down, causing the equilibrium value of π_0^e to decrease. Q.E.D.

Lemma 2. Optimal type of central banker is monotonically decreasing in term length, *T*.

Proof. Please see Appendix B.

Theorem 3. In a Model 2 economy where the central banker's type is fixed, the optimal term length of office of central banker is infinite.

Proof. The Government's optimal choice of optimal term length problem is

$$\max_T \overbrace{\frac{1}{1-\beta^T} V_T(T)}^{E_\varepsilon[W_{GOV}(T)]} \quad (45)$$

where $V_T(T)$ denotes the expected discounted utility of the Government

$$V_T(T) = \sum_{t=0}^{T-1} \beta^t E_{\varepsilon_t}\left[-\frac{A}{2}(\hat{y}_t - \bar{y})^2 - \frac{\hat{\pi}_t^2}{2} \right] \quad (46)$$

in which

$$\hat{y}_t = \frac{1}{1+\hat{\lambda}}\varepsilon_t$$

$$\hat{\pi}_t = \hat{\lambda}\bar{y} - \frac{\hat{\lambda}}{1+\hat{\lambda}}\varepsilon_t.$$

The first-order condition of the Government's problem is:

$$\frac{\partial E_\varepsilon\left[W_{GOV}(T)\right]}{\partial T} = \frac{1}{\left(1-\beta^T\right)^2}\beta^T \ln \beta V_T(T) + \frac{1}{1-\beta^T}\frac{\partial V_T(T)}{\partial T} = 0 \tag{47}$$

where

$$\begin{aligned}\frac{\partial V_T(T)}{\partial T} &= \frac{1}{1-\beta}\beta^T \ln\beta\left[\left(\frac{A+\hat{\lambda}^2}{2(1+\hat{\lambda})^2}\right)\sigma_\varepsilon^2 + \left(\frac{A+\hat{\lambda}^2}{2}\right)\bar{y}^2\right] \\ &\quad + \left(\frac{1-\beta^T}{1-\beta}\right)\frac{\partial\hat{\lambda}}{\partial T}\left[\left(\frac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\right)\sigma_\varepsilon^2 - \hat{\lambda}\bar{y}^2\right]\end{aligned} \tag{48}$$

One can rewrite the first order condition to the Government's problem as

$$\frac{\partial E_\varepsilon\left[W_{GOV}(T)\right]}{\partial T} = \frac{1}{1-\beta}\frac{\partial\hat{\lambda}}{\partial T}\left[\frac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_\varepsilon^2 - \hat{\lambda}\bar{y}^2\right] = 0 \tag{49}$$

Note that for all parameter specifications, $\frac{\partial\hat{\lambda}}{\partial T} < 0$ by Lemma 1, and $\lim_{T\to\infty}\frac{\partial\hat{\lambda}}{\partial T} = 0$. Without loss of generality, for values of σ_ε^2 which are sufficiently small relative to $\bar{y}^2$,

$$\frac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_\varepsilon^2 - \hat{\lambda}\bar{y}^2 < 0 \tag{50}$$

will hold. Also, we note that as T increases, $\frac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_\varepsilon^2$ increases, and $\hat{\lambda}\bar{y}^2$ decreases, hence as T increases the difference between these two values decreases. Hence,

$$\lim_{T\to\infty}\left[\frac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_\varepsilon^2 - \hat{\lambda}\bar{y}^2\right] = 0 \tag{51}$$

Hence, for $1 \le T \le \infty$, $\frac{\partial E_\varepsilon (W_{GOV})}{\partial T} > 0$, i.e., $E_\varepsilon [W_{GOV}(T)]$ is monotonically increasing in *T*, and $\lim_{T \to \infty} \frac{\partial E_\varepsilon (W_{GOV})}{\partial T} = 0$, then $E_\varepsilon [W_{GOV}(T)]$ reaches its maximum at $T=\infty$. Q.E.D.

This result suggests that under the condition that the central banker stays the same type as she was appointed, and given the initial conditions of the economy $A, \sigma_\varepsilon^2, \bar{y}$, and β, the Government appoints the most weight-conservative central banker to implement policy for an infinite appointment term. Below, we relax the assumption that the central banker stays the same type throughout her term.

4.2 Optimal term length when central banker shirks (general case with 0<μ<1)

Let $\hat{\lambda}_\mu, \{\hat{\pi}_t(\varepsilon_t)\}_{t=0}^{T-1}, \{\hat{y}_t(\varepsilon_t)\}_{t=0}^{T-1}$ be a given equilibrium in Model 3 with a given term length *T*, and let $V_T^\mu(T)$ denote the Government's expected discounted utility in a given term length of *T* periods, that is,

$$V_T^\mu(T) = \sum_{t=0}^{T-1} \beta^t E_{\varepsilon_t} \left\{ \mu^t \left[-\frac{A}{2}(\hat{y}_t(\varepsilon_t) - \bar{y})^2 - \frac{\hat{\pi}_t(\varepsilon_t)^2}{2} \right] + (1-\mu^t)\left[-\frac{A}{2}(\tilde{y}_t(\varepsilon_t) - \bar{y})^2 - \frac{\tilde{\pi}_t(\varepsilon_t)^2}{2} \right] \right\} \quad (52)$$

where

$$\tilde{y}_t(\varepsilon_t) = \frac{1}{1+A}\varepsilon_t \text{ and}$$

$$\tilde{\pi}_t(\varepsilon_t) = A\bar{y} - \frac{A}{1+A}\varepsilon_t$$

Government's infinite lifetime expected discounted utility is defined as follows:

$$E_\varepsilon\left[W_{GOV}^\mu(T)\right] = \sum_{\tau=0}^{\infty} \beta^{\tau T} \sum_{t=0}^{T-1} \beta^t E_{\varepsilon_t} \left\{ \mu^t \left[-\frac{A}{2}(\hat{y}_t(\varepsilon_t) - \bar{y})^2 - \frac{\hat{\pi}_t(\varepsilon_t)^2}{2} \right] + (1-\mu^t)\left[-\frac{A}{2}(\tilde{y}_t(\varepsilon_t) - \bar{y})^2 - \frac{\tilde{\pi}_t(\varepsilon_t)^2}{2} \right] \right\} \quad (53)$$

One can rewrite (53) as

$$E_{\varepsilon}\left[W^{\mu}_{GOV}(T)\right] = \frac{1}{1-\beta^{T}} V^{\mu}_{T}(T)$$

Lemma 3. Optimal type of central banker in Model 3 is monotonically decreasing in term length, *T*.

Proof. We skip the proof since it is the same as the proof of Lemma 2.

Theorem 4. In a Model 3 economy where the central banker's type arbitrarily switches to A during her term of office, the optimal term length of office is a finite number $1 < T < \infty$.

Proof. Please see Appendix D.

5. Comparative Statics Results

In this section, the effects of the exogenously given parameters, $A, \sigma^2_{\varepsilon}$, and $\bar{y}$ on the optimal choices of the Government are examined. These parameters describe the economic environment, and ultimately affect the independence of the appointed central banker. More specifically, we establish the direct effects of A and $\bar{y}$ on the optimal term length, and the effect of σ^2_{ε} on the optimal type of the central banker. The direct effect of σ^2_{ε} on the optimal term length proves to be inconclusive. The derivations are available upon request. Below, the results are introduced:

Claim 4. The optimal term length of central banker increases with A, the weight that the Government puts on output stabilization.

Intuitively, if the Government becomes more concerned about output stabilization, everything else constant, the credibility to commit to low inflation as perceived by the wage-setters, will decline. Hence, the need for a more conservative central banker to restore credibility will arise. To restore credibility, and to appoint a conservative central banker, the Government will set a long term of office for the central banker.

Claim 5. The optimal term length of central banker increases with $\bar{y}$, society's target level of output.

Increase in the socially desirable output will result in an increase in the wage-setter's expected inflation; therefore cause a distortion in the credibility of the existing policy. To restore credibility and pull the inflation down, the Government will appoint a more conservative central banker by setting a longer term length.

Claim 6. The optimal weight-conservativeness of central banker decreases with σ_ε^2, the variance of output shocks, for *T>1*.

Expecting a high σ_ε^2, the Government will require relatively more (monetary) accommodation from the central banker. Then, the Government will maximize expected discounted utility, and achieve the balance between credibility and flexibility with a relatively less conservative central banker.

6. Concluding Remarks

This paper primarily focused on the term length aspect of the institutional design of monetary policy. Most importantly, we established a clear negative relationship between the term length of office of central banker, and inflation bias. This result implies increased credibility of long-term policies. Furthermore, we obtained the optimal term length and optimal type of central banker as functions of existing economic environment. In this case, it is shown that the Government does not have to compromise time-consistent policies for optimal policies that would be implemented by an optimally weight-conservative central banker. Finally, the condition under which the Government appoints a central banker for finite or infinite term lengths was introduced. The central banker may enjoy an infinite term length if she does not shirk from the type that she was at the period of appointment. Otherwise, she will be hired for a finite term. In fact, this result suggests that in real economies, finite legal term lengths for central bankers are designed to assure minimum social welfare losses considering the probability that the central banker may shirk.

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Appendix A

Proof of Lemma 1 (Derivation of $\frac{\partial H(\pi_0^e)}{\partial T} < 0$)

Recall by (39),

$$H(\pi_0^e) \equiv \hat{\lambda}(\pi_0^e)\bar{y} = \frac{A\Gamma\bar{y}}{\Gamma + \Phi(1+\hat{\lambda})^3}$$

where

$$\Gamma \equiv \left(\frac{1-\beta^T}{1-\beta}\right)\sigma_\varepsilon^2 + (\pi_0^e + \bar{y})^2$$

and

$$\Phi \equiv \beta\left(\frac{1-\beta^{\mathrm{T-1}}}{1-\beta}\right)\bar{y}^2$$

Taking the first order derivative of *H(.)* with respect to *T*, we have

$$\frac{\partial H(\pi_0^e)}{\partial T} = \frac{A\bar{y}\dfrac{\beta^T}{1-\beta}\ln\beta(1+\hat{\lambda})^3\,\bar{y}^2\,[\,\sigma_\varepsilon^2 + (\pi_0^e + \bar{y})^2\,]}{\left[\Gamma + \Phi(1+\hat{\lambda})^3\right]^2}$$

$$< 0$$

Appendix B

Proof of Lemma 2 (Derivation of $\frac{\partial \hat{\lambda}}{\partial T}$ <0)

Recall by (36),

$$\lambda = \frac{A\Gamma}{\Gamma + \Phi(1+\lambda)^3} \equiv F(\lambda)$$

Plugging in the equilibrium value of initial inflationary expectations, $\hat{\pi}_0^e$, and taking the first order derivative of *F(.)* with respect to *T* yields

$$\frac{\partial F(\lambda)}{\partial T} = \frac{A\frac{\beta^T}{1-\beta}\ln\beta(1+\lambda)^3\,\bar{y}^2\left[\sigma_\varepsilon^2 + (\hat{\pi}_0^e + \bar{y})^2\right] + 2A(\hat{\pi}_0^e + \bar{y})\Phi(1+\lambda)^3\frac{\partial \hat{\pi}_0^e}{\partial T}}{\left[\Gamma + \Phi(1+\lambda)^3\right]^2}$$

$$< 0$$

since *ln β<0* and $\frac{\partial \hat{\pi}_0^e}{\partial T} < 0$. Hence, for every *λ*, the curve *F(λ)* shifts down, as a result, the equilibrium value of *λ* decreases.

Appendix C

Derivation of $\frac{\partial^2 \hat{\lambda}}{\partial T^2} > 0$

$$\frac{\partial^2 F(\lambda)}{\partial T^2} = \frac{1}{\left[\Gamma + \Phi(1+\lambda)^3\right]^3} \times \left\{ \left[\left(A\frac{\beta^T}{1-\beta}(\ln\beta)^2(1+\lambda)^3\bar{y}^2(\sigma_\varepsilon^2 + (\hat{\pi}_0^e + \bar{y})^2) + 2A\left(\frac{\partial \hat{\pi}_0^e}{\partial T}\right)^2 \Phi(1+\lambda)^3 \right) \right. \right.$$

$$\left. + 2A(\hat{\pi}_0^e + \bar{y})\Phi(1+\lambda)^3 \frac{\partial^2 \hat{\pi}_0^e}{\partial T^2} \right] \left[\Gamma + \Phi(1+\lambda)^3\right]$$

$$+ \left[2\frac{\beta^T}{1-\beta}\ln\beta\sigma_\varepsilon^2 + 2\frac{\beta^T}{1-\beta}\ln\beta\bar{y}^2(1+\lambda)^3 - 4(\hat{\pi}_0^e + \bar{y})\frac{\partial \hat{\pi}_0^e}{\partial T} \right]$$

$$\left. \times \left[A\frac{\beta^T}{1-\beta}\ln\beta(1+\lambda)^3(\sigma_\varepsilon^2 + (\hat{\pi}_0^e + \bar{y})^2) + 2A(\hat{\pi}_0^e + \bar{y})^2\Phi(1+\lambda)^3 \frac{\partial \hat{\pi}_0^e}{\partial T} \right] \right\}$$

$\frac{\partial^2 F(\lambda)}{\partial T^2} > 0$, since $\frac{\partial \hat{\pi}_0^e}{\partial T} < 0$ and $\frac{\partial^2 \hat{\pi}_0^e}{\partial T^2} > 0$ as

$$\frac{\partial^2 H(\hat{\pi}_0^e)}{\partial T^2} = \frac{1}{\left[\Gamma + \Phi(1+\lambda)^3\right]^4} \left[A\bar{y}\frac{\beta^T}{1-\beta}(\ln\beta)^2(1+\hat{\lambda})^3\bar{y}^2(\sigma_\varepsilon^2 + (\hat{\pi}_0^e + \bar{y})^2) \right] \left[\Gamma + \Phi(1+\hat{\lambda})^3\right]^2$$

$$+ A\bar{y}\frac{\beta^T}{1-\beta}\ln\beta(1+\hat{\lambda})^3\bar{y}(\sigma_\varepsilon^2 + (\pi_0^e + \bar{y})^2)\left[\frac{\beta^T}{1-\beta}\ln\beta(\sigma_\varepsilon^2 + \bar{y}^2)\right]$$

$$> 0$$

which implies that $\frac{\partial^2 \hat{\pi}_0^e}{\partial T^2} > 0$. Finally, we conclude that $\frac{\partial^2 F(\lambda)}{\partial T^2} > 0$ which implies that $\frac{\partial^2 \hat{\lambda}}{\partial T^2} > 0$, meaning that the equilibrium value of λ decreases at a decreasing rate-for low values of *T*, the decline in $\hat{\lambda}$ is faster, relative to the decline for high values of *T*.

Appendix D
Proof of Theorem 4

For notational convenience, we denote $\hat{\lambda}_\mu = \hat{\lambda}$ from now on.

Recall that

$$E\left[W_{GOV}^{\mu}(T)\right] = \sum_{\tau=0}^{\infty} \beta^{\tau T} V_T^{\mu}(T) = \frac{1}{1-\beta^T} V_T^{\mu}(T)$$

where

$$V_T^{\mu}(T) = \sum_{t=0}^{T-1} \beta^t \left\{ \mu^t \left[-\frac{A}{2}\left(\frac{1}{1+\hat{\lambda}}\varepsilon_t - \bar{y}\right)^2 - \frac{1}{2}\left(\hat{\lambda}\bar{y} - \frac{\hat{\lambda}}{1+\hat{\lambda}}\varepsilon_t\right)^2 \right] + (1-\mu^t)\left[-\frac{A}{2}\left(\frac{1}{1+A}\varepsilon_t - \bar{y}\right)^2 - \frac{1}{2}\left(A\bar{y} - \frac{A}{1+A}\varepsilon_t\right)^2 \right] \right\}$$

We can rewrite $V_T^{\mu}(T)$ as

$$V_T^{\mu}(T) = \left(\frac{1-(\beta\mu)^T}{1-\beta\mu}\right)\left[f(A) - f(\hat{\lambda})\right] - \left(\frac{1-\beta^T}{1-\beta}\right) f(A)$$

where

$$f(A) = (A + A^2)\frac{1}{2}\left[\frac{1}{(1+A)^2}\sigma_\varepsilon^2 + \bar{y}^2\right]$$

$$f(\hat{\lambda}) = (A + \hat{\lambda}^2)\frac{1}{2}\left[\frac{1}{(1+\hat{\lambda})^2}\sigma_\varepsilon^2 + \bar{y}^2\right]$$

Arranging, $E\left[W_{GOV}^{\mu}(T)\right]$ becomes

$$E[W_{GOV}^{\mu}(T)] = \left(\frac{1}{1-\beta^T}\right)\left(\frac{1-(\beta\mu)^T}{1-\beta\mu}\right)\left[f(A) - f(\hat{\lambda})\right] - \frac{1}{1-\beta} f(A)$$

Taking the first order derivative of $E\left[W_{GOV}^{\mu}(T)\right]$ with respect to *T*, we obtain

$$\frac{\partial E[W_{GOV}^{\mu}(T)]}{\partial T} = \frac{1}{(1-\beta\mu)(1-\beta^T)}\left[\ln\beta\frac{\beta^T(1-\beta^T)}{1-\beta^T} - \ln\mu\beta^T\mu^T\right]\left[f(A)-f(\hat{\lambda})\right]$$
$$+\frac{1-\beta^T\mu^T}{(1-\beta\mu)(1-\beta^T)}\frac{\partial\hat{\lambda}}{\partial T}\left[\frac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_{\varepsilon}^2 - \hat{\lambda}\bar{y}^2\right]$$
$$=0$$

We rewrite $\dfrac{\partial E[W_{GOV}^{\mu}(T)]}{\partial T} = 0$ as

$$(1-\beta^T\mu^T)\frac{\partial\hat{\lambda}}{\partial T}\left[\frac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_{\varepsilon}^2 - \hat{\lambda}\bar{y}^2\right] = [f(A)-f(\hat{\lambda})]\left[\ln\mu(\beta^T\mu^T) - \ln\beta\frac{\beta^T(1-\mu^T)}{1-\beta^T}\right]$$

We can rewrite the above equality as

$$\beta^T\mu^T = 1 - \frac{[f(A)-f(\hat{\lambda})][\ln\mu(\beta^T\mu^T) - \ln\beta\dfrac{\beta^T(1-\mu^T)}{1-\beta^T}]}{\dfrac{\partial\hat{\lambda}}{\partial T}\left[\dfrac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_{\varepsilon}^2 - \hat{\lambda}\bar{y}^2\right]}$$

We define

$$\beta^T\mu^T \equiv L(T)$$

and

$$1 - \frac{[f(A)-f(\hat{\lambda})]\left[\ln\mu(\beta^T\mu^T) - \ln\beta\dfrac{\beta^T(1-\mu^T)}{1-\beta^T}\right]}{\dfrac{\partial\hat{\lambda}}{\partial T}\left[\dfrac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_{\varepsilon}^2 - \hat{\lambda}\bar{y}^2\right]} \equiv K(T)$$

Taking the first order derivatives of *L*(*T*) and *K*(*T*) with respect to *T*, we have

$$\frac{\partial L(T)}{\partial T} = \beta^T\mu^T\ln\beta\mu < 0$$

and

$$\frac{\partial K(T)}{\partial T}=\frac{1}{\left\{\frac{\partial\hat{\lambda}}{\partial T}\left[\frac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_\varepsilon^2-\hat{\lambda}\bar{y}^2\right]\right\}^2}$$

$$\times\left\{-\left(\frac{\partial\hat{\lambda}}{\partial T}\right)^2\Theta\Psi^2-\frac{\partial\hat{\lambda}}{\partial T}\Psi[f(A)-f(\hat{\lambda})]\left[\ln\mu\ln\beta\mu(\beta^T\mu^T)+\ln\beta\frac{\Theta}{1-\beta^T}\right]\right.$$

$$\left.+[f(A)-f(\hat{\lambda})]\Theta\left[\frac{\partial^2\hat{\lambda}}{\partial T^2}\Psi-\left(\frac{\partial\hat{\lambda}}{\partial T}\right)^2\left[\frac{3A-2\hat{\lambda}+1}{(1+\hat{\lambda})^3}\sigma_\varepsilon^2+\bar{y}^2\right]\right]\right\}$$

where

$$\Theta=\ln\mu(\beta^T\mu^T)-\ln\beta\frac{\beta^T(1-\mu^T)}{1-\beta^T}>0;$$

$$\Psi=\frac{A-\hat{\lambda}}{(1+\hat{\lambda})^3}\sigma_\varepsilon^2-\hat{\lambda}\bar{y}^2<0$$

and $\frac{\partial\hat{\lambda}}{\partial T}<0, \frac{\partial^2\hat{\lambda}}{\partial T^2}>0$. This obtains $\frac{\partial K(T)}{\partial T}<0$. Also note that at $T=1$, $K(1)=1$, and as $T\to\infty$, $K(T)$ approaches to negative infinity. Since $L(T)$ always stays positive, and since $K(T)$ eventually becomes negative, there exists a T^* where $K(T)$ cuts $L(T)$ from above, i.e. at some $1<T^*<\infty$, $L(T^*)=K(T^*)$, which implies that $\frac{\partial E[W_{GOV}^{\mu}(T)]}{\partial T}=0$ at some finite term length, T^*.

Appendix E

Derivation of $\frac{\partial \hat{\lambda}}{\partial \mu} < 0$

Taking the first order derivative of *F* with respect to μ, we obtain

$$\frac{\partial F(\lambda)}{\partial \mu} = -A\bar{y}^3(1+\lambda)^3 \frac{1}{(1-\beta\mu)^2}\left[\beta(1-\beta^T\mu^T) - T\beta^T\mu^{T-1}(1-\beta\mu)\right]\left[\sigma_\varepsilon^2 + (\hat{\pi}_0^e + \bar{y})^2\right]$$
$$- A\Phi(1+\lambda)^3 2(\hat{\pi}_0^e + \bar{y})\frac{\partial \hat{\pi}_0^e}{\partial \mu} \times \frac{1}{\left[\Gamma + \Phi(1+\lambda)^3\right]^2}$$
$$< 0$$

in which Γ and Φ are as defined before, and $\beta(1-\beta^T\mu^T)T\beta^T\mu^{T-1}(1-\beta\mu) > 0$, and $\frac{\partial \hat{\pi}_0^e}{\partial \mu} < 0$. As μ increases, the graph of the function *F(λ)* shifts down, as a result, the equilibrium value of $\hat{\lambda}$ decreases, implying that $\frac{\partial \hat{\lambda}}{\partial \mu} < 0$.

Özet

Merkez bankası bağımsızlığı: Optimal 'Muhafazakârlık' ve optimal görev süresi

Birçok ekonomide merkez bankası başkanının görev süresi, ilgili yasal düzenleme ile bir kurala bağlanmıştır. Bununla beraber, bu kuralın etkinliği, merkez bankasının inanılabilir ve bağımsız politikalar izlemesi ile iktidardaki hükümetin bu kurala sadık kalmasına bağlıdır. Diğer taraftan, merkez bankasının izlediği politikalar, bağımsız olmalarının yanında, olası üretim şoklarına karşı yeterince esneklik gösterebilme özelliğine de sahip olmalılardır. Bu çalışmada, politikalarda inanılabilirlik ve esneklik arasındaki dengenin, hükümetin merkez bankası başkanını, toplumun uzun dönem faydasını ençoklaştıran bir optimal görev süresi için ataması ile sağlanabileceği sonucuna varılmaktadır. Atanan merkez bankası başkanının üretim stabilizasyonuna karşı enflasyon stabilizasyonuna optimal derecede ağırlık vermesine ek olarak, atandığı optimal süre boyunca 'zamana tutarlı' politikalar izliyor olmasının, inanılabilirlik ve esneklik arasındaki dengenin elde edilmesi bakımından gerekli olduğu da belirtilmektedir.